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LECTURES ON
DIELECTRIC THEORY AND INSULATION

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DIELECTRIC THEORY
AND
INSULATION

BY

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PREFACE

During the past few years, having devoted particular attention to a review of the literature of the anomalous properties of dielectrics and to experimental research in the field of insulation for high-voltage equipment, it appeared natural to select this subject for a series of lecture to be given in France, as Exchange Professor in Engineering and Applied Science. The subject is somewhat special and one which until quite recently has not attracted the interest which it deserves from electrical engineers. There is an important relation between the anomalous properties of dielectrics and the performance of high voltage insulation.

The literature of the subject reveals an enormous quantity of experimental results and attempts at theoretical analysis. There is, however, the widest divergence of point of view and almost complete absence of a common ground. As a consequence it seemed to me well worth while to attempt some coordination and to bring together in some form the more important results of all this work. My efforts in this direction have been given during the past few years in a course of lectures to advanced students. The nine lectures of the present volume cover the same ground but in somewhat condensed form. It is hoped that the reader will realize the necessity of such condensation in a series of public lectures, and will understand many glaring omissions and short cuts which could never be tolerated in a finished treatise.

The first lecture reviews briefly the more important postulates of the classical theory of perfect dielectrics. Its object is to indicate the point of view of the writer and to establish a common ground for the lectures which follow. The material of the seven following lectures is taken almost entirely from the literature of physics and electrical engineering. In some of the more important instances the results of various investigators have been given in some detail. As a consequence these pages contain little that is new. I hope, however, that my efforts to coordinate the diverse material will be found useful by others who are working in this difficult but stimulating field. Moreover, I have ventured

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to make a few criticisms and comparisons and to point out the directions in which further research will probably prove profitable. I have indicated also some of the results of my own researches on composite insulating materials subjected to high values of alternating stress. The last lecture is devoted entirely to this subject. The bibliography divided into several general subjects includes all of the works that have been consulted and many others.

J. B. Whitehead.

Baltimore, Md.
April, 1927.

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DIELECTRIC THEORY AND INSULATION

LECTURE I

ELEMENTARY THEORY OF PERFECT DIELECTRICS

Of the various component parts entering into electrical systems of all characters, the insulation is the least susceptible to exact computation and design. Our knowledge of the behavior of insulating materials under variations of voltage, frequency, temperature, and mechanical stress is very meager. Our methods of control of the important properties of insulating materials, singly and in combination, are very imperfectly developed. The insulation is the most vulnerable element of every electrical system, its deterioration is the most rapid, and the length of life of the system is that of its insulation. As a consequence, in the design of insulation liberal provision must be made against these uncertainties; large factors of safety must be allowed, thus resulting in increased size and cost.

The important electrical properties of insulation are resistivity, specific inductive capacity, dielectric phase difference, and dielectric strength. While the relative importance of these properties may vary as amongst different types of service, in many instances all of them are involved, as for example, in all types of high-voltage alternating-current service.

The classical theory of dielectrics coming to us from Faraday and Maxwell assumes that dielectrics have only the one property of specific inductive capacity. It tells us nothing of dielectric strength, nothing of phase difference, or power factor, under alternating stress, and very little of the conductivity of dielectrics. Yet this is the theory as developed in all textbooks on physics, except a few of advanced type. It is therefore clearly insufficient to guide us in our efforts to understand and control the important properties of insulation as utilized in electrical engineering equipment. It is in view of this failure of the accepted formal theory of dielectrics to account for the important dielectric phenomena as observed in the insulation of electrical engineering equipment that I have chosen as topic for this series of lectures, "Dielectric Theory and Insulation." In them we shall review the salient features of dielectric behavior for which the classical theory fails to account, review briefly the literature describing the vast amount of experimental research in the field of dielectric behavior, bring out of it such coordination and agreement with fundamental theory as may be possible, and finally indicate some of the directions in which further experimental research may be expected to lessen the confusion and uncertainty now existing.

OUTLINE OF FUNDAMENTAL DIELECTRIC THEORY

The beginnings of the classical theory of dielectrics, as set forth in modern textbooks of physics, may be said to have begun with the work of Franklin, Cavendish, and Faraday. Franklin, in 1748, had shown the importance of the dielectric medium to the properties of the Leyden jar which had been announced shortly before. Cavendish was apparently impressed by Franklin's work; he compared electrostatic capacities and instituted a number of experiments in which he sought for different values of induced charges as amongst various substances. Apparently Cavendish did not appreciate the distinction between conductors and dielectrics. In effect his experiments measured the relative capacities of condensers having different dielectrics, the values being compared to the capacity of a standard air condenser. He gives a list of numerical values of these ratios. This list is thus a series of values of K, the dielectric constant, although Cavendish did not suggest the name nor emphasize specific inductive capacity as a property of the medium. Faraday's elaborate studies of the exact equality between inducing and induced charges on concentric conductors, with and without dielectrics and conductors between them, led to his famous conception of a point-to-point transfer throughout the medium of the influence of an electric charge. Following immediately therefrom are his well-known ideas of lines of force, tubes of induction, and dielectric polarization. Faraday's great contributions in this field were his emphasis of the importance of the medium in the phenomenon in electrostatic induction, his measurements on different dielectrics, and in effect his discovery and announcement of the fundamental property, specific inductive capacity. Maxwell's matchless genius appreciated the importance of Faraday's work, and building upon it he developed the theory of potential, as applied to the electric field and to dielectric behavior, which is still universally accepted by physicists and which always constitutes the starting point of any discussion of dielectric phenomena. In order that we may follow intelligently the important work which has been done in recent years in the field of insulation and the efforts which have been made to link up the behavior of insulation with the fundamental theory of dielectrics, it will be necessary for us to glance briefly at a few of the outstanding relations based on that theory.

Electric Potential.—We review first the idea of electric potential, and in tracing the origin of our present conception we find, at the very foundations, the name of two distinguished French physicists, Coulomb, 1723–1806, and Poisson, 1781–1840. The former with his torsion balance gives us the law of attraction and repulsion between point charges. The theory of potential as developed by Poisson for the magnetic case was utilized by Maxwell in his complete development for the electric case. From Coulomb's expression for the force between isolated charges in air,

$$f = \frac{qq'}{r^2} \tag{1}$$

we arrive at the definition of unit charge and unit current on the basis of the electrostatic system of units, and also the value of the electric force at a distance r from a point charge q

$$X = \frac{q}{r^2} \tag{2}$$

in which X has both intensity and direction, and hence is a vector quantity.

If q is positive and a unit positive charge $+1$ be placed at A (Fig. 1), the unit positive charge will be acted upon by a mechanical force X, and a length of line AX may be selected to represent the value and the direction of this mechanical force.

If the charge $+1$ moves along any path from A to B, work will be done by or against the force due to q. If r' is greater than r, it is readily seen that the total mechanical work V, expended by the electric field due to q, on the charge $+1$ as it moves in any path between A and B, is positive and of the value

$$V = q\left(\frac{1}{r} - \frac{1}{r'}\right) \tag{3}$$

If the point B is removed to an infinite distance, we have

$$V_A = \frac{q}{r};\ V_A = \sum \frac{q}{r} \tag{4}$$

Thus the work expended by the system when the charge $+1$ moves from point A to a very great distance is dependent only on the charge q and the distance of A from q. It is therefore a property of the point A and is called the "potential" at A. We may therefore designate it as V_A. Furthermore if there are other charges in the neighborhood, each of them will exert its force at A, and the total potential at A will be the sum of the potentials due to the separate charges, *i.e.*, as indicated in the second form of Eq. 4.

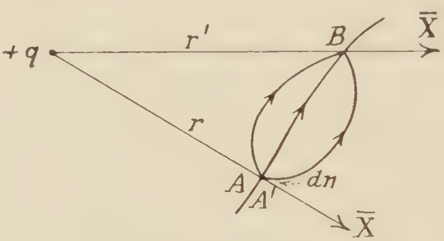

FIG. 1.—Electric potential.

Let $V_A + dV_A$ represent the potential at a neighboring point on the line AX at distance dn from A measured in the direction AX. The difference in potential between A and A' will therefore be $V_A - (V_A + dV_A) = Xdn$

or

$$X = -\frac{dV}{dn} \tag{5}$$

This is a very important and useful relation. It states that the electric force is always in the direction of decreasing potential and that at any point the force in any direction is equal to the space rate of the decrease of the potential in that direction.

Difference of Potential.—It will be noted that Eq. (3) for the work done as the charge $+1$ moves from A to B is also an expression for the difference of potential between the points A

and B. Thus we say that both potential and difference of potential are of the nature of mechanical work. This fundamental idea originating in the simple electrostatic case is of such importance that we may well pause a moment to extend it to the more common case of power in any electric circuit, for it is here that we find the bridge between the unknown nature of electric phenomena and the methods we have adopted for expressing their laws in terms of the more familiar mechanical units of force, energy, and work.

If the charge $+1$ moves from A to B, the work done by the electric field is $V_A - V_B$. If a charge $+1$ moves from A to B in every second, the rate at which work is done by the system is $V_A - V_B$ per second. If a charge Q moves from A to B every second, the system does work at the rate $Q\ (V_A - V_B)$ per second. Now the current is the number of units of charge passing any point in the circuit per second. Thus in the case above the work done per second, or the power, is $Q(V_A - V_B)$, in which Q is the current and $V_A - V_B$ the difference of potential. We need not concern ourselves with the nature of the system which causes the difference of potential between the points A and B. If therefore we can imagine two terminals from any source of constant difference of potential, placed at A and B, and a conductor permitting the flow of Q units of charge per second connecting points A and B, we have all of the essential elements for explaining the nature of the energy flowing in electrodynamic circuits in terms of the fundamental notion of potential difference.

Potential of a Conductor.—As A (Fig. 1) approaches q, V_A increases. If q is uniformly distributed over a sphere of radius ρ, then the potential at a point on the surface of the sphere becomes $\frac{q}{\rho}$, and the sphere is found to have a potential due to its own charge. If there are other charged conductors in the neighborhood, each will contribute to the potential of the sphere; that is, the potential of a charged conductor is due not only to its own charge, but also to the influence of any other nearby charges.

Electrostatic Induction.—Remembering that all points on a conductor on which the charges are stationary are necessarily at the same potential, let us consider two parallel conducting plates as indicated in Fig. 2. Let the upper plate be at a constant potential V_1 and the lower plate at potential $V_0 = 0$. The potential of either plate is due to the combined influence of its own charge and the charge on the other plate. The plate V_1 with its positive charge, therefore, tends to raise the potential of the lower plate. But the lower plate remains at zero potential $V_0 = 0$. To offset the tendency of the upper to raise the potential of the lower plate, therefore, a negative charge must appear on the latter. This negative charge must obviously be greater and greater as the upper plate approaches the lower. The corresponding tendency of the negative charge of the lower plate to lower the potential of the upper plate is attended by a similar increase in the positive charge of the upper plate. Thus by bringing two conductors of large area near together the values of their charges may be quite large even at a small difference of potential. Such an arrangement of conductors is known as a condenser, and the charge on either plate resulting from unit difference of potential between them is the "capacity" of the condenser.

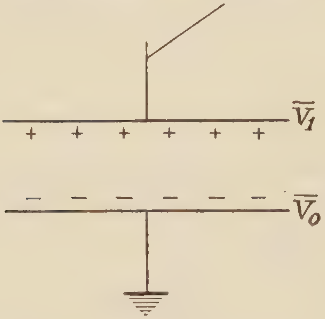

Fig. 2.—Electrostatic induction.

Specific Inductive Capacity.—We must now introduce into our expression a factor representing the influence of a dielectric medium other than air. Cavendish found that, if certain materials were introduced between the parallel plates of Fig. 2, the charges on the plates were increased. Such materials are known as dielectrics, and Faraday has given the name of "specific inductive capacity" to this property of the dielectric material.

Faraday defines specific inductive capacity as the ratio of the charge appearing on one plate of the condenser, when the dielectric substance is between the plates, to the charge appearing on the same plate when air is between plates, the difference of potential remaining the same in the two cases. For many materials at constant temperature this ratio is constant over a wide range of potential gradient, and it is generally considered a definite physical constant of the dielectric material.

This definition leads to a very important modification of the laws of electrostatic attraction and electrostatic potential, as derived above. If we let

$$C = \frac{Q}{V}$$

be the capacity of a condenser with air as dielectric, and

$$C' = \frac{Q'}{V'}$$

be the capacity with a dielectric K, then we have

$$\frac{C'}{C} = K = \frac{Q'V}{Q\,V'}$$

If $V = V'$, then $Q' = KQ$, in accord with the definition of K. However, if $Q = Q'$, then

$$V' = \frac{V}{K}$$

and we see that in changing from air as dielectric to a medium K as dielectric, a given set of charges on the plates may be maintained at potential difference only $\frac{1}{K}$ times the former values; or more generally, if the medium surrounding a given set of charges is changed so that the specific inductive capacities before and after the change are K_1 and K_2, then potentials, potential gradients, and forces are changed in the ratio:

$$\frac{V_2}{V_1} = \frac{F_2}{F_1} = \frac{K_1}{K_2} \tag{6}$$

In accordance with this principle we now see that Eq. (1) may be modified so as to include the influence of a medium of specific inductive capacity K by writing it in the form of

$$f = \frac{qq'}{Kr^2} \tag{7}$$

Gauss' Theorem.—A very important and useful theorem is that of Gauss. By Eq. (7) the force at a distance r due to a point charge q is

$$f = \frac{q}{Kr^2}$$

Consider any imaginary closed surface or portion of spaces surrounding q, and let N be the normal component outwards, of the force due to q, at any point P on the imaginary surface. Also let ds be the area of a small portion of the surface at the point P. Then by Fig. 3

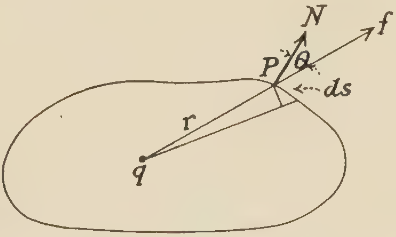

FIG. 3.—Gauss' theorem.

$$Nds = \frac{q}{Kr^2}\cos\theta \cdot ds$$

$$\iint KNds = 4\pi q \tag{8}$$

The product KN is called the "normal induction" across the surface at the point P. The theorem states that the surface integral of the normal induction outwards taken over any closed surface containing the charge q is equal to $4\pi q$. It is obvious that if there are any number of charges inside the surface, the theorem is extended to the form

$$\iint KNds = 4\pi\sum q \tag{9}$$

By a proper choice of the location of a Gauss surface, the theorem offers a simple method for proving a number of important relations. We will briefly review a few of them, keeping before us Faraday's familiar conceptions of lines of force, tubes of force, electrostatic induction ($=KF$), and tubes of induction as aids to our mental picture of the respective problems.

a. Tube of Force.—Select as the imaginary Gauss surface the portion of a tube of force formed by its sides and by two right sections of area S_1 and S_2 respectively, and let the values of electric force at S_1 S_2 be F_1 and F_2 respectively. Then the surface integral of the normal component of the force taken over the whole surface is $F_2S_2 - F_1S_1$; there is no force normal to the sides of the tube, since these sides are bounded throughout by lines of force. Moreover, since there is no charge inside the tube of force, the above expression vanishes, by Gauss' theorem, and therefore $F_1S_1 = F_2S_2$ for any two cross-sections of a tube of force in air.

b. Electric Force Normal to a Surface.—In Fig. 4 let V_1 be an electric conductor carrying a charge σ per square centimeter. Consider a Gauss surface of cylindrical form with the sides of the cylinder parallel to the lines of force and the ends of the cylinder being one in the body of the conductor and one outside the conductor and close to its surface. The total charge within this cylinder is σS, S being the cross-sectional area of the cylinder. The normal induction outwards from the surface of the cylinder is zero on the sides and over the end of the cylinder which lies within the conductor, since the sides are parallel to the electric force, and there is no force within the body of a conductor; over the remaining end of the cylinder the integral of the normal induction is the same as that over the surface V_1. And if F is the electric force normal to the surface V_1, we have by Gauss' theorem

$$\iint KNds = KFS = 4\pi\sigma S$$

and

$$F = \frac{4\pi\sigma}{K}$$

is the value of the force just outside a surface carrying a charge per unit area. Obviously if the adjacent medium is air, the value of force is $4\pi\sigma$.

c. Capacity of Parallel Plates.—From case (b) we readily derive the capacity of an area S of two parallel plates. If the plates are of wide extent, the lines of force between the plates are parallel to each other, *i.e.*, the electric force between them is uniform; thus in Fig. 4

$$F = 4\pi\frac{\sigma}{K}$$

throughout the whole region between the plates; therefore the difference of potential between the plates:

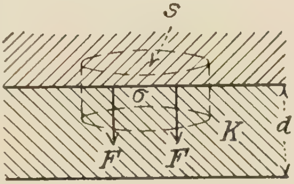

FIG. 4.—Electric force normal to a conductor.

$$V_1 - V_2 = Fd = 4\pi\frac{\sigma d}{K}$$

Thus C the capacity per square centimeter of the parallel plates is given by

$$C = \frac{\sigma}{V_1 - V_2} = \frac{K}{4\pi d}$$

d. Radial Intensity of Cylindrical Conductor.—Suppose that we have a long cylindrical conductor carrying a charge q per unit length as in a single conductor cable. We may readily find the potential gradient at any point in the region surrounding the central conductor by selecting a cylindrical Gauss surface coaxial with the conductor and with ends at right angles to the cental axis. Over such a surface of radius of r, indicated in Fig. 5, there is no force parallel to the axis of the conductor, and we have

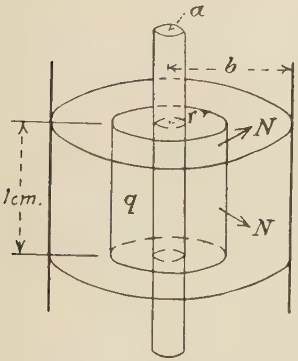

FIG. 5.—Potential gradient near a cylindrical conductor.

$$\iint KNds = 2\pi rKN = 4\pi q$$

$$\therefore \quad N = \frac{2q}{Kr} \qquad (10)$$

where K is the specific inductive capacity of the medium and N is the radial electric intensity at distance r from the axis.

e. Capacity of Two Concentric Cylinders.—If in Fig. 5 the inner cylinder has a radius a and it is surrounded by an outer cylindrical conductor of radius b, then the difference of potential between a and b is given by

$$V_a - V_b = \int_a^b N dr = \frac{2q}{K}\int_a^b \frac{dr}{r} = \frac{2q}{K} \log \frac{b}{a}$$

and therefore the capacity per unit length of coaxial cylinders is

$$C = \frac{q}{V_a - V_b} = \frac{K}{2 \log \frac{b}{a}} \tag{11}$$

Equation (11) permits us to express q in terms of the potential difference and the capacity per unit length. Substituting in Eq. (1), we obtain the radial electric stress at any point in the medium between the central conductor and the outer shield.

$$N = \frac{V}{r \log \frac{b}{a}} \tag{12}$$

The most important application of these principles is found in single-conductor high-voltage cables. The values of potential gradient in the insulation of such cables has an important influence on their life. Equation (12) shows that if b is constant, the value of the potential gradient at distance r from the axis of the conductor is independent of the specific inductive capacity and increases continuously from the outer conducting sheet to the surface of the inner conductor. The value of the maximum gradient at the surface of the inner conductor depends on the ratio $b : a$, and it may readily be shown that it is a minimum for $b = 2.73a$.

Boundary Conditions.—In many of the cases arising in practice, lines of force and tubes of induction pass from one dielectric to another. It is of importance to know the distribution of the field in such a case. We consider, first, the conditions when the field is parallel to the surface of separation and, second, when the field is normal to this surface.

a. Field Parallel to Surface of Separation.—Let K_1 and K_2 (Fig. 6) be the dielectric constants of the two media, and F_1 and F_2 the fields in each. Draw two equipotential surfaces A and B

through the surface of separation and at an infinitesimal distance dl apart, then A and B are parallel to each other, and the potential difference $V_A - V_B$ is F_1dl for the first medium and F_2dl for the second medium. Since these two are equal, $F_1 = F_2$, and our first boundary condition is that the tangential components of the electric intensity are the same on the two sides of the surface of separation.

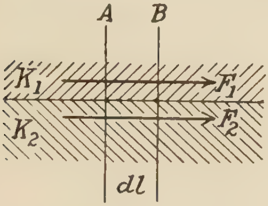

Fig. 6.—Conditions at surface of separation of two dielectrics.

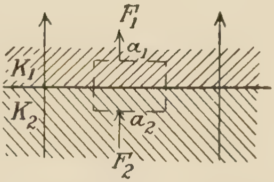

Fig. 7.—Conditions at surface of separation of two dielectrics.

b. Field Normal to Surface of Separation.—In this case we take a small closed surface with ends paralled to the boundary and sides normal to it, and apply Gauss' theorem to this surface, remembering that there is no charge on the surface of separation (Fig. 7).

$$\int\!\!\int N ds = K_1F_1a_1 - K_2F_2a_2 = 4\pi q = 0$$

whence, since $a_1 = a_2$, we have

$$K_1F_1 = K_2F_2$$

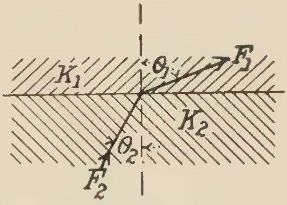

Fig. 8.—Conditions at surface of separation of two dielectrics.

and thus the second boundary condition is that at any point in the surface of separation the normal component of the induction is the same in both media. Now imagine these two field distributions to be superposed, one over the other. Each will produce a component of electric intensity at any point independently of the other. Since the parallel components are equal and vertical components are not so, the resultant lines of force and tubes of induction are bent or refracted in crossing the surface of separation. The conditions will be as represented in Fig. 8 in which the two boundary conditions are

$$F_1 \sin \theta_1 = F_2 \sin \theta_2$$
$$K_1F_1 \cos \theta_1 = K_2F_2 \cos \theta_2$$

whence we have

$$\frac{\tan \theta_1}{\tan \theta_2} = \frac{K_1}{K_2} \tag{13}$$

Also since the parallel components are in the same direction in each medium, the forces F_1 and F_2 are in the same plane normal to the surface of separation.

Extending the above, consider a small tube of force passing across the surface of separation of two media K_1 and K_2 and intercepting on that surface an area s. Let s_1 and s_2 be the areas of the right sections of the tube on the two sides of the surface of separation, then, by the above, we have

$$K_1F_1 \cos \theta_1 = K_2F_2 \cos \theta_2$$
$$s_1 = s \cos \theta_1$$
$$s_2 = s \cos \theta_2$$

Therefore $$K_1F_1s_1 = K_2F_2s_2 \tag{14}$$

Equation (14) states that throughout the length of a tube of induction, the value of the product of the induction KF and the area of the cross-section of the tube is a constant. This constant value is determined by the charge from which the tube of induction starts, as follows:

Let the tube start from area s of conductor surface and let the density of surface charge be σ. The electric intensity just outside s is

$$F = 4\frac{\pi\sigma}{K}$$

whence, we have, e being the total charge on s,

$$KFs = 4\pi\sigma s = 4\pi e \tag{15}$$

the value shown to be constant throughout the length of the tube. By a simple application of Gauss' theorem it may be shown that the tube of induction begins and ends on equal and opposite charges.

It may readily be seen from the foregoing that any equipotential surface in an electric field may be replaced by a conducting sheet if the surface density of charge at all points on the sheet is given by the relation KFs = constant = $4\pi\sigma s$. This is Green's theorem.

Energy in the Medium.—If Green's theorem be applied to a number of right sections of a tube of induction, we may imagine the tube to be made up of a number of condensers connected in series. The energy of charge of a condenser is

$$\frac{1}{2}e(V_1 - V_2)$$

where e is the charge on either plate. Since e is the same for all of the condensers forming our tube of induction, we find that the total energy is distributed along the tube in amounts which are proportional to the potential differences between the successive right sections.

Consider an elementary length dn of the tube of cross-section s. The energy in the length dn is

$$-\frac{1}{2}e\frac{dV}{dn}dn = \frac{1}{2}eFdn$$

but

$$KFS = 4\pi e$$

Therefore the energy w in the volume sdn is

$$w = \frac{KF^2S}{8\pi}dn$$

and the energy W per cubic centimeter is

$$W = \frac{KF^2}{8\pi} \tag{16}$$

Dielectric Displacement.—In the foregoing treatment we have developed our ideas of electric force and electrostatic induction by analyzing phenomena which take place in the air. These phenomena take place equally well in vacuum. They are therefore as perfect examples as we can conceive of action at a distance. The idea of action at a distance was repugnant to Faraday, and this led to his proposal that electric action takes place from point to point through the medium. Since however the electric forces manifest themselves in free space, Maxwell recognized that the ether must possess properties permitting the propagation of electric forces. He therefore pictured an elastic ether which is strained under the action of electric stress. He assumed the strain as proportional to the stress.

Under this view a difficulty is encountered in attempting to coordinate with it the behavior of solid dielectrics. We have seen that the introduction of a solid dielectric into an electric field increases the charges on conductors, if the latter are maintained at constant potential, or reduces the potentials and forces between conductors, if their charges remain constant. Maxwell accounts for these phenomena by assuming that the ether is the medium of the transfer of electric forces but that the elastic properties of the ether are changed in the presence of matter. In accordance with his view the presence of a dielectric in an electric field reduces the elastic resistance of the ether, permitting a greater displacement under a given force. Further, the total displacement is always proportional to the electric intensity and to the specific inductive capacity K.

We must imagine, therefore, according to Maxwell, that every tube of induction is in a state of electric strain which results in the presence of equal and opposite charges on the two ends. Throughout the tube, at all points positive and negative charges are "displaced" with reference to each other. The numerical value of this *displacement* is based on the relation $KFS = 4\pi e$, which as we have already noticed has a constant value throughout the length of a tube of induction. Maxwell's definition of the dielectric displacement at any point is that it is equal to the charge displaced per square centimeter normal to the tube of induction. Thus by Eq. (14) the displacement is:

$$D = \frac{e}{S} = \frac{KF}{4\pi} \tag{17}$$

Extending these ideas further, Maxwell pointed out that any time variation of the electric intensity would be accompanied by a corresponding change in the displacement and that this change in the displacement constitutes a current in the dielectric. Moreover, he assumes that this displacement current has the same magnetic effect as would an equal current in a conductor. We need not follow Maxwell further in the development of his classical equations of the electromagnetic field, as they have no direct bearing on the problems of the behavior of dielectrics in their function as insulating materials. We may note in passing, however, that Maxwell recognized that most dielectrics possess electric conductivity. He takes due account of this in his discussion of dielectrics, as we shall see.

Dielectric Polarization.—Many writers and investigators have found difficulty in Maxwell's view that the ether is the seat of electric phenomena and that its properties are affected by the presence of matter. They have preferred, therefore, to separate the influence of the ether from that of the matter. In accordance with their view, electric force is propagated in the ether. When a dielectric is placed in such a field of force, positive and negative charges contained in the ultimate particles of the medium (either molecules or atoms) change their relative positions by a limited amount proportional to the intensity of the electric field. The dielectric is assumed to become polarized in much the same way that magnetic materials become polarized under the action of magnetic fields. As a result, surface charges of opposite sign appear on the opposite faces of the dielectric, just as poles appear on magnetic materials. There is much to be said in favor of this view. It enables one to separate the influence of the ether from that of the dielectric and to attribute definite properties to the latter. If all dielectrics were perfect dielectrics, there would be no special advantages in introducing the idea of polarization, because in such dielectrics polarization is only present when an electric force is acting, disappearing as soon as the force is removed. We shall see, however, in considering the anomalous properties of dielectrics, that there are important instances when the polarization of a dielectric persists after the removal of the electric force. A number of investigators of these anomalous properties have preferred to deal with them in terms of the polarization.

In view of these two points of view it is necessary to note the relation between Maxwell's *displacement* D and the *polarization* I. The polarization is generally assumed to be proportional to the electric intensity F, by a constant of the material k, known as the "dielectric susceptibility." Its relation to the displacement is obtained by noting that the total displacement must be equal to the displacement in the ether plus the polarization of the medium. Thus we have, for a square centimeter normal to a tube of induction,

$$D = \frac{KF}{4\pi} = \frac{F}{4\pi} + kF \tag{18}$$

and consequently

$$K = 1 + 4\pi k \tag{19}$$

which gives the relation between the specific inductive capacity and the dielectric susceptibility. The equation is seen to be identical with that for the corresponding phenomenon in magnetism.

It is interesting to note in passing that the idea of a polarization of the dielectric material is in accord with the indications of modern physical investigation as to the nature of the ultimate structure of matter. We know that the atoms of all matter contain electrons, or negative charges, and that these are in motion. It is easy, therefore, to picture a movement of these charges under an external electric field in such a way as to cause the atom to take up a polarized condition. Some difficulty is encountered when we remember that the electrons are supposed to be in a rapid state of motion in closed orbits. This means that a polarization of the atom would mean a distortion or stretching of the orbits of the electrons so that on the whole the average positions of all the electrons would be shifted away from that of the positive nucleus. Other pictures have been drawn of the atom which do not involve the orbital motion of electrons. Indeed some writers have pictured a stable condition of the electrons within the atom. The presence of the electron, however, is certain, and this is a sufficient fact to account for polarization. Unfortunately, modern physics has given very little attention to the questions of the nature of specific inductive capacity and of the anomalous properties of dielectrics, which we are about to discuss. Such attention, however, as has been devoted to them always pictures polarization as due to a shift of the electron away from the normal position or orbit, as they exist in the neutral atom.

It is obvious that we have discussed the classical development of the electrostatic field only in briefest possible outline. The elementary relations which we have deduced are sufficient, however, to enable us to discuss the distribution of electric stress and electric induction in practically all of the cases which arise in the design, the testing, and the performance of dielectrics as used for electric insulation.

In our next lecture we shall describe some of the anomalous properties of dielectrics.

LECTURE II

THE NORMAL AND THE ANOMALOUS PROPERTIES OF DIELECTRICS

The Normal Properties of Dielectrics.—The outstanding features of the elementary theory of dielectric behavior outlined in the foregoing lecture are:

a. The idea of a continuous polarization of the dielectric medium as proposed by Faraday.

b. The extension of this idea by Maxwell to that of dielectric displacement.

c. The elementary development of the relations between electric force, potential, charge, displacement, and polarization.

The treatment thus built up assumes that under constant external conditions the value of the dielectric susceptibility k and of the specific inductive capacity K of a dielectric substance are constant; that is, the polarization and the displacement are rigorously proportional to the electric field intensity. This view is held widely today. It is adopted in all textbooks on the physical theory of electricity and magnetism except a very few of advanced type. It defines completely the behavior of perfect dielectrics. It is quite sufficient for the accurate computation of the distribution of the electric intensity and dielectric displacement in all cases in which the electric field has a constant value over a long period of time. For example, the simple principles laid down in Lecture I may be used for the computation of the capacities of all types of condenser, for the potential gradient in high-voltage underground cables, for the stresses in all forms of high-voltage insulation, and other similar cases.

The Anomalous Properties of Dielectrics.—It has been recognized for a long time, however, that these simple assumptions and relations are not sufficient to account for all the phenomena of dielectric behavior. For example, many dielectrics possess electric conductivity, and this conductivity is subject to wide variation. Many condensers do not take up their full charge until sometime after the application of voltage and require some time for complete discharge on short circuit. Dielectrics subjected to alternating stress absorb energy and reveal a power factor far greater than can be accounted for by their conductivities. Liquid dielectrics possess all these characteristics. The laws controlling them are very irregular and little understood.

Since these properties are not accounted for by the theory of perfect dielectrics, they have been called the "anomalous properties" of dielectrics. Moreover, they are treated but rarely in textbooks and find place only in those of advanced type such as Winkelmann, Graetz, and your own Bouasse. The obvious suggestion is that the anomalous properties are unusual and of small importance. For a long time this estimate was perhaps warranted, because they assumed no importance in the practical applications of dielectrics. In the enormous expansion of the field of electrical engineering in recent years, however, and the wide variety of the forms in which dielectrics are used as insulation, it was soon recognized that in these fields these anomalous properties are perhaps the most important of all, owing to the limitations they impose on the performance of insulation. Two examples will serve to illustrate this and, at the same time emphasize the two most important anomalous properties.

Dielectric Absorption.—The first example is the phenomenon of dielectric absorption or viscosity. It is by virtue of this property that a condenser under continuous potential difference does not take up its charge instantaneously. Charge continues to flow into the condenser over a period of time sometimes very long. Moreover, on short circuit the condenser does not discharge instantaneously. Time is required for complete loss of charge. After short circuit the condenser may accumulate a considerable residual charge.

Dielectric absorption was recognized in the very earliest days of electrostatic experiment. The Leyden jar (1746) was chiefly remarkable for its manifestation of residual charge. It is described at some length in the experiments of Benjamin Franklin (1748). It was thus well recognized by Faraday and Maxwell. In fact, as is well known, Maxwell gives us the first attempt ever made at a theoretical explanation of the phenomenon of absorption. We shall review this theory in some detail in a later lecture. It is not entirely satisfactory, and since Maxwell's time there have been many studies of the property of absorption. For the most part these have been made by physicists, and for a long time absorption apparently has been considered only an interesting anomalous property of dielectrics, of interest chiefly in its bearing on the theories of the nature of dielectric behavior. In recent years absorption has assumed importance in its bearing on the performance of the insulation of telegraph and telephone cables. In this case it causes attenuation and energy loss, which result in the blurring of signals and speech, and in the reduction of distances of transmission. While improvements have been made, this character of trouble has persisted in communication engineering throughout the whole extent of its modern development. A further manifestation of dielectric absorption is found in the measurement of all types of insulation resistance. As is well known, considerable time must elapse before the current comes to a steady value under continuous voltage, and as a consequence in such measurements it is necessary to specify the interval of time, after the application of voltage, at which the current is read.

Dielectric Loss.—The second conspicuous anomalous phenomenon in dielectric behavior is the energy loss in solid dielectrics when subjected to alternating electric stress. This loss was noted by Siemens, in 1864, in his attempts to use static condensers in alternating circuits. This loss is much greater than that due to the conductivity of the dielectric. It heats the internal structure of high-voltage insulation, thereby increasing its conductivity, and often leads to breakdown. The nature of this loss is not completely understood. For some time it was generally considered that it is of the nature of a molecular hysteresis similar to that in magnetic materials. This view is quite prevalent still. We shall see, however, that there are serious objections to this hypothesis.

It is my purpose in the following lectures (*a*) to describe the conspicuous manifestations and experimental observations of the phenomenon of dielectric absorption; (*b*) to present the principal theories which have been advanced to explain it; (*c*) to describe the phenomenon of dielectric loss; (*d*) to present evidence that dielectric loss is in large measure due to dielectric absorption, so that these two anomalous properties probably arise in the same causes; and (*e*) to discuss the behavior of the more conspicuous forms of high-voltage insulation in the light of our present knowledge of absorption and dielectric loss.

DIELECTRIC ABSORPTION—DESCRIPTION AND DEFINITION

We may conveniently describe the phenomenon of dielectric absorption and other allied properties of dielectrics in terms of the behavior of a simple parallel-plate condenser. In accordance with fundamental theory, in which a dielectric is considered as subject to polarization but free from conductivity, if a constant potential difference E_0 is applied at time $t = 0$ to the terminals of the condenser, there flows in the connections a "normal charging" current $i_1(t)$ which varies with the time. The time variation of this normal charging current may be determined experimentally or may be computed from the well-known differential equation:

$$L\frac{d^2i_1}{dt^2} + R\frac{di_1}{dt} + \frac{i_1}{C} = 0 \qquad (1)$$

L and R being respectively the self-induction and resistance of the connections, and C the capacity of the condenser as computed from its dimensions and the specific inductive capacity of the dielectric. The solution of the equation shows that $i_1(t)$ is either a periodic function with decreasing amplitude, or decreases to zero with aperiodic amplitude, according to well-known relations between L, R, and C. With the usual small values of L and R, it always results that the normal charging process is practically ended after a very small fraction of a second.

It is probable, however, that no dielectric or insulator exists which is entirely free from conductivity. Maxwell recognized this and, as is well known, his general equations for the electromagnetic field include terms for both conduction and displacement currents. He assumes that conductivity and dielectric displacement occur simultaneously and, as it were, in parallel. The conduction and displacement currents are superposed or combined. We must therefore extend our definition of a normal dielectric to include also the property of conductivity, and in the above case of a parallel-plate condenser we must recognize the conduction current i_g, added to the charging current $i_1(t)$. The value of i_g will be determined by the instantaneous value of the potential difference on the plates of the condenser and by the dimensions and specific conductivity of the dielectric. In this view, when the condenser is discharged by short circuit, the difference of potential at its terminals is zero, and there is no conduction current in the dielectric. Thus the normal discharge current i_2 is the displacement current of dielectrics and follows the same laws as i_1 but is in opposite direction, that is, $i_2(t) = -i_1(t)$. We shall see below that there are some difficulties in assigning definite values to the conductivities of many dielectrics, and that as a consequence i_g may not obey the simple laws of conduction. We shall discuss this matter in a later lecture.

The presence of conductivity in the dielectric still permits us to compute theoretically the variation of the charge and discharge currents of a normal condenser, for we have added only a resistance term, this resistance being connected in parallel with the condenser. When we come to experimental observation, however, we find that the theoretical behavior is found in only very few cases. These for the most part are limited to condensers with air or gas as dielectric. In the cases of fluid and solid dielectrics, we find marked departures from theory. The charge and discharge currents not only show the initial sharp decrease called for by the equation, ending within a very short interval of time, but these currents persist as so-called *residual* currents, in some cases for days or even for months. The charge and discharge currents due to the change in the displacement in the dielectric should die out in extremely short time intervals. In experiment, however, it is found that these currents merge into the relatively much smaller, more slowly diminishing residual

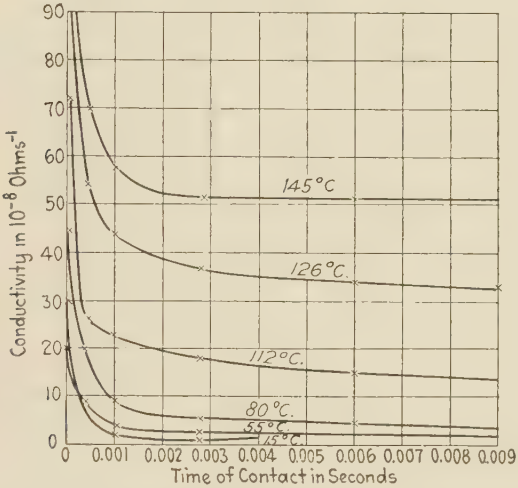

FIG. 9.—Dielectric absorption in glass. (*Hopkinson.*)

currents. Hopkinson found that for a certain flint-glass condenser 97 per cent of its total charge was discharged in 0.000059 second, and he states that Sir William Thompson found a similar result. Based on experiments of this character it is now generally assumed that a condenser has a constant value of capacity, and its dielectric a definite dielectric constant for initial brief intervals of charge and discharge. This capacity is generally referred to as the geometric capacity of the condenser. Thus even in dielectrics which show the residual charge and discharge currents mentioned above, the dielectric constant corresponding to rapid charge has a definite value, although the residual currents may show wide variation among several samples of the same material. We shall see later that in the alternating case the apparent capacity of a condenser with increasing frequency approaches the geometric capacity as a limiting value.

These residual or anomalous currents occur during both charge and discharge. We will designate them $i_1^1(t)$ and $i_2^1(t)$ respectively. The total currents on charge and discharge are thus given by

$$\begin{aligned} I_1(t) &= i_1(t) + i_g + i_1^1(t) \\ I_2(t) &= i_2(t) + i_2^1(t) \end{aligned} \tag{2}$$

In many dielectrics the simple relation $i_2^1(t) = -i_2^1(t)$ obtains; that is, that both in magnitude and in type of variation with time, anomalous charge and discharge currents are identical. On the other hand, there are cases in which this relationship does not exist; the anomalous charging current being greater than the discharge current. Accordingly, von Schweidler distinguishes two types of anomalous current: (*a*) *the irreversible anomalous current* in which case $i_2^1(t)$ vanishes, and (*b*) *the reversible anomalous current* for which we have the relation $i_2^1(t) = i_1^1(t)$, as in the case for the normal displacement current.

It usually happens in the case of electrical insulation that both types of anomalous current are present. Various materials show the several types of both normal and anomalous current in different amounts. Granier has proposed another classification of dielectrics based on their behavior as regards *discharge* as follows:

a. Dielectrics which discharge their total charge instantaneously. Examples: air, sulphur.

b. Dielectrics which discharge their total charge but only over a period of time. Examples: certain samples of mica and ebonite.

c. Dielectrics which on discharge give up only a portion of the charges which they have absorbed on charging.

This classification does not emphasize the fact that the conductivity in dielectrics is of two types: one, best observed in liquids but also in some solids, not obeying Ohm's law; and the other, found in solids only and following Ohm's law. With this addition the classification is in accord with that of von Schweidler. Apparently there are few, if any, dielectrics having conductivity and no residual effects.

In view of the foregoing we may return to our elementary condenser with solid dielectric. When continuous voltage is applied, we have three components of current; $i_1(t)$ representing the charging of the geometric capacity; the conduction current i_g which may or may not obey Ohm's law; and a slowly decaying, anomalous absorption, or residual current $i_1^1(t)$. If this anomalous current $i_1^1(t)$ is all of the reversible type, it builds up the stored charge

$$Q_1 = \int_0^t i_1^1(t) \cdot dt$$

which may be completely recovered on a long-time short circuit of the condenser. If, however, the condenser is discharged during only a brief interval of short circuit and then allowed to stand open, part of the stored charge Q_1 may appear as a building up of a difference of potential between the plates. A further *residual* discharge may then be obtained. In such cases, if the total discharge is equal to the original charge, that is, if $Q_2 = Q_1$, the total charge is reversible, and $i_1^1(t)$, the current on charge, is called the *reversible absorption current.* In liquid dielectrics there is also a steady decreasing residual current on charge of the type $i_1^1(t)$. This current does not in general obey the same laws as in the case of solids. In nearly all such cases there is no corresponding discharge current $i_2^1(t)$. In these cases we call $i_1^1(t)$ the *irreversible absorption current.*

DIELECTRIC ABSORPTION AS OBSERVED IN EXPERIMENT

The Reversible Absorption Current.—This is the absorption current commonly observed in the best and purest solid insulators. There is good experimental evidence that the charge and discharge currents have the same form; that is, in Eq. (2) $i_1^1(t) = i_2^1(t)$. Systematic and reliable studies of the mode of variation of the absorption current may be said to have begun with Kohlrausch in 1854. They received a great stimulus in the experiments of Sir John Hopkinson in the period 1876–1897. Hopkinson appears to have had the benefits of Maxwell's advice (Paper 19). As we have seen, Maxwell himself was deeply interested in the phenomena of dielectrics and treats them at great length.

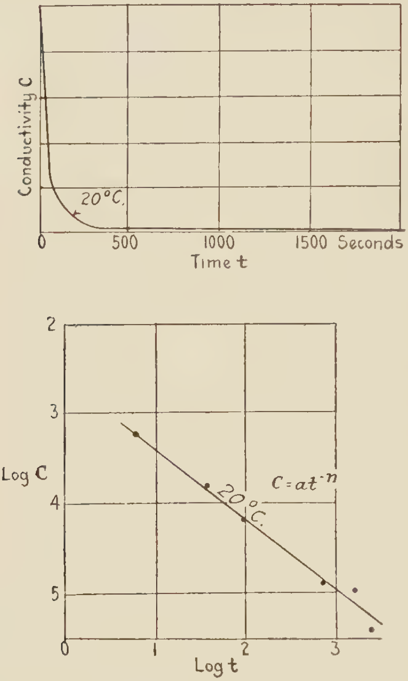

Fig. 10.—Dielectric absorption in mica. (*J. Curie.*)

We see in Figs. 9, 10, and 11 absorption curves taken from the papers of Hopkinson and J. Curie and one taken by the author on a high grade of composite insulation.

In each we see the characteristic curve of the reversible absorption current. It begins at high value, decreases at first rapidly, then more slowly, and eventually tends asymptotically to a final steady value. The shape of the curve immediately suggests either a negative logarithmic or a negative exponential relation between current and time. In fact both of these relations have been found on analysis of the curves of experiment.

Kohlrausch, Hopkinson, Giese, J. Curie, von Schweidler, Shuddemagen, Jordan, Tank, and others have found that the time variation of the reversible absorption current may be expressed with close accuracy by the empirical formula

$$i_1^1(t) = at^{-m} \tag{3}$$

Others have found that the following formulae express their observations more closely:

J. Curie, Malclés, Wagner, Steinmetz:

$$i_1^1(t) = ae^{-bt^n} \tag{4}$$

J. Curie, von Schweidler, H. A. Wilson:

$$i_1^1(t) = \frac{a}{1 + bt} \tag{5}$$

Trouton and Russ:

$$i_1^1(t) = \frac{a}{b + t} + c \tag{6}$$

Fleming and Dyke, Thornton:

$$i_1^1(t) = ae^{-bt} + c \tag{7}$$

Among these various formulae, Eq. (3) appears to express the results of experiment in a single term better than any of the others. On the other hand, it has often been noted that by using the sum of several terms of the form of Eq. (4) all with $n = 1$ and with different values of the other constants, an expression may be written agreeing with any set of observations (Hopkinson, von Schweidler, Wagner, Steinmetz). Probably a similar result could be found using Eq. (3). When a single expression is needed in discussions of theory, Eqs. (3), (4), and (5) are used (Tank, Bouasse), sometimes with two or three terms but rarely more. Maxwell's theory of dielectric absorption explains the exponential form of Eq. (4), and so the latter is generally used by supporters of Maxwell's theory. Several observers (Fleming

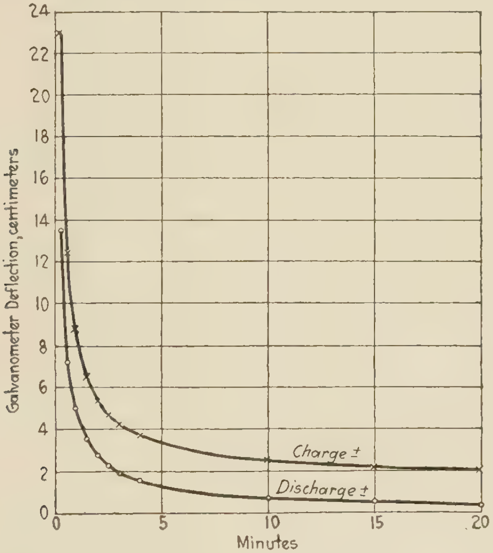

FIG. 11.—Dielectric absorption in a composite insulation. (*Whitehead.*)

and Dyke, Thornton, Schreiber) have noted that the charging current curve has a region of linear change with the time, gradually changing into an asymptotic decrease.

It should be noted, however, that for very short intervals of time, *i.e.*, as t approaches zero, Eq. (3) indicates a current approaching infinite value which is manifestly impossible. The other formulae indicate constant initial values of $i_1^1(t)$ at $t = 0$; they therefore make a stronger appeal. The initial portion of the curve represents the merging of the decaying charging current of the geometric capacity with the reversible absorption current. Their separation through experiment has apparently never been attempted. Such a study should prove of considerable interest and value.

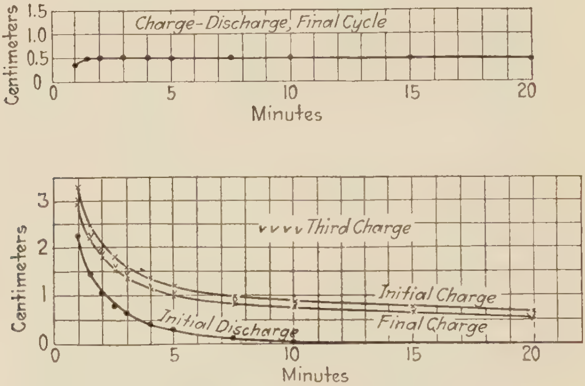

FIG. 12.—Conductivity of a composite insulator.

The final steady value of the reversible absorption current is to be considered as pure conduction current i_g and may be used for computing the specific conductivity of the dielectric. Figure 12 shows the curves of discharge, and it is seen that the difference between charge and discharge curves has in some cases a constant value. This relation is an excellent test for the pure reversible character of the absorption current. We shall see curves of a quite different appearance when discussing the irreversible absorption current.

General Formula.—With change in the value of the applied voltage or of the thickness of the dielectric, the time rate of variation of the current is unchanged, but the values are increased in proportion to the voltage gradient (Curie, Wilson). The stationary final value of the charging current also obeys Ohm's law in most cases and so may be considered as a normal conduction current. In a few cases, this conduction current is either absent or negligibly small, although the absorption may be quite large, *e.g.*, ebonite (Gaugain, Malcles). Generally, therefore, we may write

$$i_1^1(t) = B \cdot \mathrm{CE}_0\varphi(t) \tag{8}$$

in which B is a constant, $\varphi\ (t)$ a continuously diminishing definite function for the material of the dielectric, and C the geometric capacity proportional to the area and inversely as the thickness.

Temperature has a marked influence on the absorption current. Hopkinson was one of the earliest investigators, and he showed clearly that the absorption in glass increases with the temperature. It has been generally supposed from the work of von Schweidler, Wagner, Zeleny, and others that in Eq. (8) B increases with the temperature but that the form of the function $\varphi\ (t)$ suffers little or no change. Lately Wagner ("Die Isolierstoffe der Elektrotechnik," Schering), proceeding from Maxwell's theory of a layer dielectric (see below) and observations on a paper condenser, concludes that the effect of temperature is to increase the rate of decay of $i_1^1(t)$, at the same time increasing its initial values so that the amount of absorbed charge is unaffected. Granier notes the same effect, but further evidence is needed to support the generality of this conclusion. There is universal agreement, however, that temperature increases both the currents $i_1^1(t)$ and $i_2^1(t)$, as well as the final conduction current.

Other Facts.—Other facts noted in connection with $i_1^1(t)$, some of which need further study, are the following: Some substances, *e.g.*, ebonite (Gaugain, Malcles) show notable residual or absorption currents, and no final conduction current. In these cases, if the electrodes are separated from the dielectric, the final charge on the electrodes is less than if an equal thickness of metal is substituted for the dielectric, indicating that absorption is not due to conductivity alone. The absorbed charge is not removed by exposure to radioactive substances. A dielectric withdrawn from its electrodes affects an electroscope, but shows zero total charge in a Faraday cylinder, thus indicating a polarized condition.

The Irreversible Absorption Current.—In Eq. (2) the component i_g of the charging current is pure conduction current and therefore is not present when the condenser is discharged by short circuit.

It is therefore irreversible, and flows only on charging. The irreversible absorption current also flows only on charging, but differs from i_g in that it varies with the time, decreasing usually very slowly (often expressed $i_1^1(t) = b \cdot t^{-m}$) from a definite initial value. Its final constant value is reached earlier and earlier as the voltage is higher and higher. This final constant value may not be considered as a normal conduction current since it does not obey Ohm's law. With different values of E the final value of the current is not proportional to E but generally increases more slowly than the voltage. In some cases an apparent saturation current is observed, and on the other hand in a very few instances the final current appears to increase more rapidly than in proportion to the voltage. This type of irreversible absorption current is observed particularly in liquid dielectrics. Sometimes, however, it is noted in composite solids, as for example built-up laminated insulation, impregnated paper, etc., particularly in the undried state (see Fig. 13). Experimental investigations of this current in liquids are numerous, but very few in the case of solids, beyond the simple observation of a difference between the values and shapes of the currents of charge and discharge. A notable exception is the work of Evershed on the influence of moisture on the conductivity of paper. His results show several of the properties of the irreversible absorption current as mentioned above, and are reviewed in greater detail in another paragraph. The general trend of the conclusions from the studies on liquids is

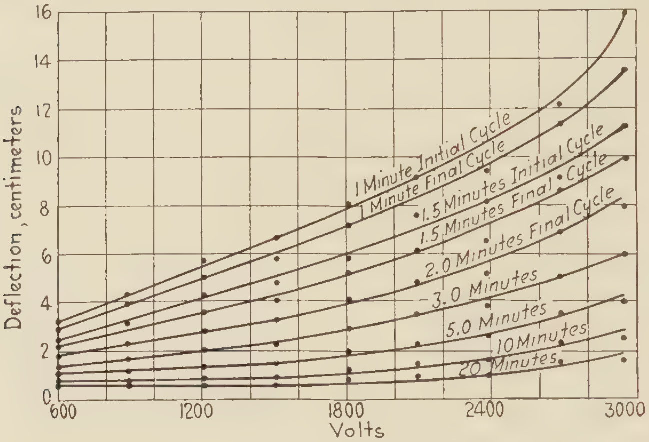

FIG. 13.—Irreversible absorption current in solid insulators.

that this current may be due in part to dissociated impurities (sometimes eliminated permanently by prolonged flow of current), but more generally to ions which are more or less free to move, and which obey the laws of gaseous conduction. A more extended description of the phenomena in liquids appears to lie outside our present purpose of discussing the phenomena in commoner forms of electrical insulation. It may be stated, however, that so far as the irreversible absorption current in solids is concerned, there is good evidence that it is associated with the presence of moisture, or of impurities giving it an electrolytic character, and that with the progressive elimination of both, this type of current is reduced to very small proportions, if not completely eliminated. Some further discussion of this type of conduction will be found in the paragraph dealing with anomalous conductivity as a theory in explanation of absorption. We leave the subject, therefore, with a list of references in the bibliography.

The Principle of Superposition.—One of the earliest features noticed in the phenomenon of residual charge was the occasional reversal of its sign as related to the foregoing charge. Hopkinson (Papers 18 and 19) studied this property in glass at great length. Applying a succession of charges and discharges at irregular intervals and polarities, he showed that the discharge curve had widely different forms, and frequent reversals of polarity as related to the type and succession of the foregoing charging periods. He

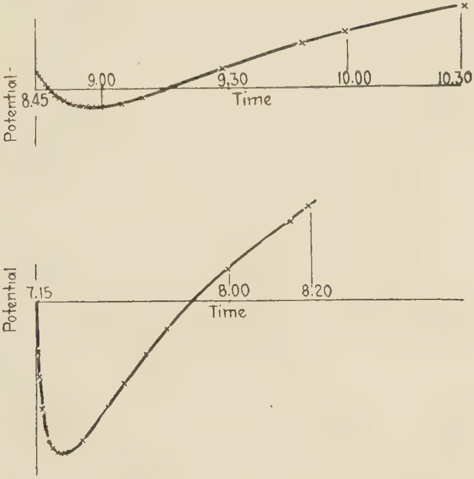

FIG. 14.—Reversal of residual charge. (*Hopkinson.*)

quotes Sir William Thomson (Lord Kelvin) as saying: "The charges come out of the glass in the inverse order in which they go in," and develops an equation for residual potential analogous to the equation for the angular twist of a torsion thread under a succession of couples of various magnitudes and direction. Each couple, applied for some time, is supposed to cause a slow time yield, *i.e.*, increase of angular displacement, and the decay of these is reflected in subsequent twisting couples. He states that this view was suggested to him by Maxwell; it is essentially the same as used by Boltzmann for the "afterworking" of any mechanical strain. From many experiments Hopkinson concludes that his equations are only roughly approximated, but that "it seems safe to infer that the effects on a dielectric of past and present electric forces are superposable." Figure 14 shows one of Hopkinson's curves.

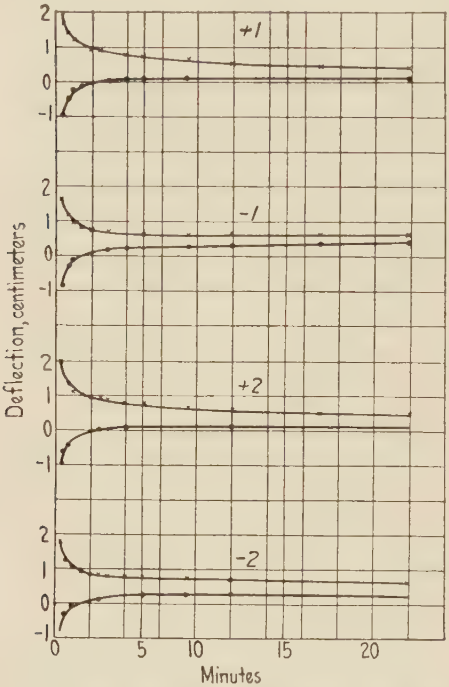

Fig. 15.—Reversal of residual charge; composite insulation.

The principle thus proposed by Hopkinson has been studied experimentally by a number of investigators and has been confirmed beyond serious question. Figure 15 shows a curve observed on composite insulation, by the author. We may note particularly the careful and accurate studies by J. Curie of the specific inductive capacities and the conductivities of quartz, a number of other crystals, and of mica, sulphur, gypsum, ebonite, porcelain, etc. Working with a piezoelectric cell as a balancing source of charge, he developed a zero method permitting the measurement of the absorption current from 0.1 second after the application of the charging potential. Three brief laws announced by Curie are worth repeating, although the third dealing with the principle of superposition appears somewhat inadequate:

1. *Loi de Proportionnalité aux Forces Électromotrices.*—Les ordonnées de la courbe des intensités du courant de charge en fonction du temps sont rigoureusement proportionnelles à la force électromotrice.

2. *Loi des Épaisseurs.*—Pour une même force électromotrice, les ordonnées de la courbe des intensités du courant de charge sont en raison inverse de l'épaisseur de la lame diélectrique.

3. *Loi de Superposition.*—Chaque variation de force électromotrice entre les deux faces de la lame agit comme si elle était seule.

The following more exact statement of the principle of superposition is due to von Schweidler: After any change ΔE in the voltage, the current as observed takes on an additional term and is then made up of a term representing the undisturbed variation of the original current, and a superposed current which so varies as though a voltage of the absolute value of the change in voltage ΔE had been applied to the uncharged condenser. If, therefore, at time $t = 0$, the voltage E_0 is applied, and then at times t_1, t_2, t_3, etc., any positive or negative changes of voltages $\Delta_1 E$, $\Delta_2 E$, $\Delta_3 E$ are applied, then we have as the value of i_1^1 at any instant t.

$$i_1^1(t) = BC[E_0\varphi(t) + \Delta_1 E \cdot \varphi(t - t_1) + \Delta_2 E \cdot \varphi(t - t_2) + \dots] \quad (9)$$

Obviously the conduction current is not included in the above expression. Thus, if the final conduction current is subtracted from the charging current, it is seen that the equality of the reversible charge and discharge currents ($i_1^1 = i_2^1$) is merely a special case of the principle of superposition.

Still more generally it follows, if the principle of superposition be true, that for any continuous variation of the voltage as expressed by the function $E(t)$, the reversible absorption current is given by

$$i_1{}^1(t) = BC \cdot \int_0^t \frac{d}{du}[E(u)] \cdot \varphi(t - u)du \quad (10)$$

in which t is the instant at which $i_1^1(t)$ is measured and u is the elapsed time controlling the variation of E.

Your own Professor Bouasse, in his "Cours de Magnetisme et d'Électricité," has discussed the principles of superposition with great clearness. He emphasizes the fact that it is an abstract theory and that it rests on two hypotheses:

a. Le corps se souvient pendant un certain temps des modifications subies; ce souvenir s'efface suivant une certain fonction decroissante du temps $\varphi(\tau)$, que nous appellerons *"facteur d'oubli."*

Nous pourrons essayer comme facteur d'oubli, par exemple, une exponentielle $ae^{-b\tau}$, ou une fonction de la forme $1 : (b + \tau^n)$; en particulier, l'expérience suggère le facteur $1 : (b + \tau)$.

b. La modification eprouvée sous l'influence d'une déformation est indépendante des déformations antérieures: c'est en cela même que consiste *le principe de la superposition.*

Ces hypothèses sont tres générales; montrons-en l'application à la polarisation diélectrique sous l'influence d'un champ uniforme.

L'effet est alors l'intensite I de polarisation, la cause est le champ électrique P.

Au temps τ(compté vers le passe, à *partir de l'epoque actuelle prise pour origine*) et pendant le temps $d\tau$, le champ etait $P(\tau)$. Nous posons que la polarisation actuelle I est dueau champ actuel augmenté de l'action (plus ou moins oubliée) des champs antérieurs.

On a donc: $I = k\left[P + \int_0^\infty P(\tau) \cdot \varphi(\tau) \cdot d\tau.\right]$

Exprimons $P(\tau)$ en fonction du temps t. Le phènomène qui se passe au temps τ dans le passé à partir du temps t actuel, se passe effectivement au temps absolu $t - \tau$.

En définitive nous écrirons au temps t:

$$I = k\left[P + \int_0^\infty P(t - \tau) \cdot \varphi(\tau) \cdot d\tau.\right] \tag{11}$$

La fonction φ doit être nulle pour $\tau = \infty$, avoir une valeur finie pour $\tau = 0$. Nous ferons nos calculs en posant, suivant les cas:

$$\varphi(\tau) = \frac{a}{b + \tau}, \qquad \varphi(\tau) = ae^{-b\tau}$$

Equations (10) and (11) readily permit analysis of the rate of loss of charge on an open condenser, after brief charge, or after short circuit following a long charge, as investigated experimentally by Kohlrausch, and analytically in other manner by Maxwell. Their greatest value and importance, however, are found in the means they offer for computing currents, capacities, phase differences, and losses, in many cases of insulation, subjected to alternating stress. We review this application in a later lecture.

The bibliography contains reference to the important work of many others, but in studying the phenomenon of absorption one cannot afford to omit any part of the beautifully simple papers of J. Curie.

In the following lecture we shall discuss some of the theories which have been advanced to explain the anomalous properties of dielectrics.

LECTURE III

THEORIES OF DIELECTRIC PHENOMENA

MAXWELL'S THEORY OF ABSORPTION

Specific Inductive Capacity.—Explanations and theories are relative terms only. In saying that we have a theory for a new natural phenomenon we usually mean that we can explain it in terms of a more fundamental phenomenon whose laws are well recognized. Quite often, however, these fundamental phenomena are not themselves understood. Thus, since the earliest discovery of the properties of dielectrics, many explanations of their origin and nature have been suggested. Few, if any, of these suggestions, however, have risen to the dignity of a theory which can be supported by quantitative experimental test.

Specific inductive capacity is one of the simplest of electrical phenomena. As we have already indicated, Faraday showed the importance of the dielectric medium in the phenomenon of electrostatic induction. Thus there was introduced the idea of a separation of the charges of the ultimate particles of an insulating medium by an inductive action of the electrodes. This is known as polarization, and the internal separated charges contribute to the total force. These ideas are sufficient to explain all normal dielectric properties and are the basis on which the classical development of dielectric theory by Maxwell rests. The development and consequences of Maxwell's theory are so well known as to need no comment here. It may be mentioned, however, that so far as the nature of specific inductive capacity is concerned, Maxwell pictured an electrically elastic ether pervading all space and matter. The electric field displaces the medium, the displacement is opposed by the elastic forces, and these latter are different in the presence of molecules of matter, thus explaining the varying dielectric properties of the latter.

It is obvious that Maxwell's picture of an elastic ether with matter suspended in it is not a very definite one. Certainly it cannot be described as a theory. Ever since the discovery of the electron and our increasing knowledge of the structure of the atom, physicists have been attracted to the question of the origin of the property of specific inductive capacity in dielectrics. Indeed, before the discovery of the electron, Helmholz in 1870 proposed that the molecules contain charged particles which are drawn out of their natural positions under the influence of an external field. Recent studies in this direction picture the displacement of electrons within the atom, or the elongation of the orbit of an electron resulting in an electric doublet, as called for by the early ideas of Faraday and Maxwell. It is not my purpose to discuss these studies and the researches which have been carried out in efforts to support them. Questions of this character lie rather within the field of the theoretical physicist. It may be stated, however, that little progress has been made in the attempt to find a rational explanation of the phenomenon of specific inductive capacity. On the other hand there is no present suggestion that the fundamental classical theory as already outlined will need any modification in so far as specific inductive capacity is concerned.

Theories of Dielectric Absorption.—The fact that many dielectrics possess a measurable conductivity was recognized early, and Maxwell treats conductivity as a common property of dielectrics. He assumes that dielectric displacement and conductivity are contemporaneous phenomena, and he developed his theory for dielectrics possessing the property of conductivity as well. The conductivity is treated as though it were similar to that in metallic conductors. There is no suggestion as to its origin nor to the laws of its variation. We know, however, that so far as a definite specific conductivity, similar to that in metallic conductors, is concerned, it is probable that no dielectric possesses it. Laws governing the variation in the two cases are quite different, as we shall see.

Thus, even in the matters of the so-called *normal properties* of dielectrics, we have very little certain fundamental theory upon which to rely. Modern physical theory has added little or nothing to the ideas of Maxwell's time. Nevertheless it is a fact that ever since the phenomenon of residual charge was recognized, there have appeared suggestions and hypotheses as to the general nature of the underlying processes. Beginning with mere suggestions in the early days, as for example, the slow penetration or "soaking in" of the charge (readily shown to be untenable), they have increased in elaboration and complexity as further knowledge of the phenomenon has been gained. In reviewing these theories today, we are presented with such pictures as a viscous yielding of the dielectric, frictional motion of molecules and electrons, infinitesimal conducting particles embedded in insulating sheaths, the free motion of electrolytic ions, dielectric hysteresis, the capillary motion of water, etc. It is impossible within a limited space to give a complete view of all these theories. It appears best, therefore, to attempt their classification into a few groups and to give one or two conspicuous examples in each group, with some attempt to outline the reasoning and evidence in support.

Most physicists offering explanations of absorption have apparently considered that the observed phenomena in anomalous dielectrics are not consistent with the fundamental equations of the electrostatic field, and have built up other equations based on new special properties of the dielectric, not embraced in the older theory. We shall see that in nearly every instance these are merely general hypotheses or ingenious mathematical expressions which do no more than present other descriptions of the phenomena. There is, however, one conspicuous instance, that of Maxwell, in which the fundamental equations are taken as a basis. We may, therefore, select two of our groups in accordance with these two views, and add a third to include those looking to present theories of the internal structure of the atom as bases for the explanation of dielectric behavior. There is also a possible fourth group as proposed by von Schweidler, in which anomalies of conductivity are invoked, but the evidence in support of this group is not so strong as in the others.

Our classification of theories of dielectric absorption then is as follows:

1. Those in which the fundamental electric equations are retained, and the anomalies of dielectric behavior are attributed to anomalies of the structure of the dielectric medium.

2. Those in which the departures from the fundamental laws are attributed to anomalies of dielectric displacement without reference to the nature of the underlying mechanism. Dielectric displacement is not proportional to field strength but dependent on the preceding state of the dielectric.

3. Explanation of displacement and its anomalies traced to the motion of electrons within the atom.

4. Explanations based on anomalies of conductivity, such as the free motion of ions, electrolytic dissociation, water in bulk or in capillary filaments.

GROUP 1

Maxwell.—All students of dielectric theory are familiar with Maxwell's treatment of absorption. He starts with the assumption that all dielectrics have both specific inductive capacity and conductivity as we know them in normal dielectrics, and that under electric force they function simultaneously and independently of each other. The assumption is justified by the experimental facts that conductivity may be observed in even the best insulators, and that poor insulators with very high conductivity also manifest specific inductive capacity. No further assumptions, as for example, as to the origin of these properties, are necessary in Maxwell's development. He first explores the question as to whether a normal dielectric, that is, one possessing specific inductive capacity and conductivity, can of itself manifest the characteristic phenomenon of absorption. Making use of Poisson's equation $\Delta^2 V + 4\pi\rho = 0$ and the equation of continuity of current flow, he shows that in a homogeneous dielectric no residual charge can appear. He also investigates the behavior of a condenser with single normal dielectric when charged, allowed to stand open, and then discharged, a cycle which always shows absorption in an anomalous dielectric. He finds that none should appear in the case of the condenser with normal dielectric. We need not follow him through these preliminary steps. They are of importance only as showing that conductivity alone in a dielectric cannot account for the phenomenon of absorption. It is important, however, that we follow him in outline at least in the development of his theory of the behavior of a *composite* dielectric.

Maxwell shows that a dielectric which consists of a mixture of two or more different materials, even though under our closest examination it may appear to be homogeneous, may show the phenomena of absorption. The dielectric, therefore, is to be thought of as a conglomeration or mixture of different normal dielectrics, that is, dielectrics having in general different values of K and λ (specific conductivity). He shows that a variation of the values of these constants, among the several constituent materials, is sufficient to account for dielectric absorption, but that if the dielectric has the same values of K and λ throughout, absorption cannot appear.

For simplicity and to permit mathematical treatment, Maxwell considers a parallel plate condenser containing layers of different component substances, all parallel to each other and normal to the electric field. In order to explain the fundamental reasoning, let us consider unit area parallel to the electrodes of a condenser having only two of these layers of thicknesses a_1 and a_2. On the application of a difference of potential E_0 to the terminals of the condenser, we have the following sequence of phenomena:

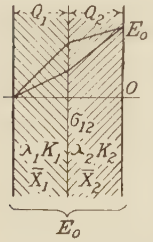

Fig. 16.—Maxwell's layer condenser.

1. At the first instant the potential gradients X_1 and X_2 in the two layers are in the inverse ratio of their dielectric constants K_1 and K_2 and the displacement D_1 and D_2 are equal, or (see Fig. 16).

$$D_1 = \frac{K_1 X_1}{4\pi} = D_2 = \frac{K_2 X_2}{4\pi} \tag{1}$$

2. As the result of the above difference in the potential gradients and also the difference in the values of the conductances λ_1 and λ_2, the conduction currents p_1 and p_2 are different in the two layers.

3. Therefore a charge σ_{12} collects at the surface of separation.

4. The process continues until the currents in the two media are equal and the potential gradients gradually assume values determined by the values of λ_1 and λ_2. The displacements are no longer equal, and their difference is accounted for by the charge σ_{12} on the surface of separation.

It is an essential assumption of Maxwell's treatment that the total current in each dielectric in this transient period is made up at all times of the sum of the conduction current and the displacement current. The latter is the time rate of change of the displacement, that is,

$$i = i_1 = p_1 + \frac{d}{dt}(D_1) \tag{2}$$

$$i = i_2 = p_2 + \frac{d}{dt}(D_2)$$

Thus we have at all times between the initial and final states

$$\sigma_{12} = D_2 - D_1 \tag{3}$$

and

$$\frac{d}{dt}(\sigma_{12}) = p_1 - p_2 = \frac{d}{dt}(D_2) - \frac{d}{dt}(D_1) \tag{4}$$

in which i is the current in the external circuit. These relations hold for a dielectric of any number of layers. Substituting the values of p and D, we have for any layer n

$$i = X_n\lambda_n + \frac{K_n}{4\pi}\frac{dX_n}{dt}; \tag{5}$$

from which the value of X_n may be found in terms of i. Similarly the expressions for the gradients in other layers may be obtained. Combining them all, the value of the total difference of potential

$$(E_0 = (a_1X_1 + a_2X_2 + \ . \ . \ . \ )) \tag{6}$$

in terms of the current is derived. In the general case of n substances, having different values of the ratio $(\lambda : K)$, Maxwell tells us that when the unwieldly expression for the terminal electromotive force is cleared, there results a linear differential equation of the nth order with respect to E_0, the terminal electromotive force, and of the $(n - 1)$th order with respect to i. Maxwell does not give us this equation, and the above statement is all that we have from him as to the form of the function showing the variation of the absorption current with the time. However the exact and complete expression for the current has recently been derived by F. D. Murnaghan, who gives several other interesting consequences of Maxwell's theory.

It is a simple matter to investigate this question for the case of two layers only. This has been given in convenient form by K. W. Wagner[1] as follows:

On first applying the potential difference E_0 we have as the initial condition in the two dielectric layers, each of thickness d,

$$D_1 = \frac{K_1X_1}{4\pi} = D_2 = \frac{K_2X_2}{4\pi} \tag{7}$$

and

$$X_1d + X_2d = E_0 \tag{8}$$

whence

$$X_1 = \frac{E_0K_2}{d(K_1 + K_2)}; \ X_2 = \frac{E_0K_1}{d(K_1 + K_2)} \tag{9}$$

These two values of X_1 and X_2 will in general result in different values of the conduction currents in the two layers. The charge σ_{12} on the surface of separation increases until the conduction currents become equal. D_1 and D_2 then have different values.

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[1] Dr. K. W. Wagner's treatment appears in the opening chapter of H. Schering's "Die Isolierstoffe der Elektrotechnik," Julius Springer, Berlin.

In this final steady state, the values of X_1 and X_2 are determined by the conductivities, and we have

$$p_1 = \lambda_1 X_1 = p_2 = \lambda_2 X_2 \tag{10}$$

whence

$$X_1 = \frac{E_0}{d}\frac{\lambda_2}{(\lambda_1 + \lambda_2)} \text{ and } X_2 = \frac{E_0}{d}\frac{\lambda_1}{(\lambda_1 + \lambda_2)} \tag{11}$$

It is during the passage from the first to the second of these states that we see a gradual decrease in the difference between the conduction currents in the two layers. The charge σ_{12} accumulating at the surface of separation is the "absorbed" charge. The essential condition during this interval, according to Maxwell, is given by

$$\frac{d}{dt}(\sigma_{12}) = p_1 - p_2 = \frac{d}{dt}(D_2 - D_1) \tag{12}$$

The equation states that the difference in the conduction currents is in every instance equal to the rate of change of the difference in the dielectric displacement. In other words, the total current in each layer is the sum of the conduction and displacement currents, and the total current has the same value in each layer. Substituting, we have

$$X_1\lambda_1 - X_2\lambda_2 = \frac{1}{4\pi}\left(K_2\frac{dX_2}{dt} - K_1\frac{dX_1}{dt}\right) \tag{13}$$

and from Eq. (8)

$$\begin{aligned} \frac{dX_1}{dt}\frac{(K_1 + K_2)}{4\pi} + X_1(\lambda_1 + \lambda_2) &= \frac{E_0}{d}\lambda_2 \\ \frac{dX_2}{dt}\frac{(K_1 + K_2)}{4\pi} + X_2(\lambda_1 + \lambda_2) &= \frac{E_0\lambda_1}{d} \end{aligned} \tag{14}$$

The solution of the first of these equations is

$$X_1 = \frac{\lambda_2}{\lambda_1 + \lambda_2}\frac{E_0}{d} + Ce^{-\frac{t}{T}} \tag{15}$$

in which C is a constant of integration and

$$T = \frac{K_1 + K_2}{4\pi(\lambda_1 + \lambda_2)} \tag{16}$$

Murnaghan's general analysis already referred to, shows for two layers:

$$T = \frac{\alpha_1 + \alpha_2}{\alpha_1 b_2 + \alpha_2 b_1}; \qquad \text{where} \qquad \begin{aligned} \alpha_1 &= 4\pi a_1 K_1 = 4\pi\frac{a_1}{K_1} \\ b_1 &= 4\pi K_1\lambda_1 = 4\pi\frac{\lambda_1}{K_1} \end{aligned}$$

$$T = \frac{a_{12}K_2 + a_2K_1}{4\pi(a_1\lambda_1 + a_2\lambda_1)} \tag{16a}$$

This reduces to Eq. (16) when $a_1 = a_2$. It is interesting, as giving the value of T for layers of different thickness.

As bearing on the influence of small amounts of impurities in causing absorption, note that as a_2 approaches 0, T approaches $\frac{K}{\lambda_1}$; that is, for a very thin layer T may still be quite large.

However, the coefficient of the exponential term is:

$$A_1 = \frac{\alpha_1\alpha_2(b_1 - b_2)^2}{(\alpha_1 b_2 + \alpha_2 b_1)(\alpha_1 + \alpha_2)^2} = \frac{\frac{a_1}{K_1}\frac{a_2}{K_2}\left(\frac{\lambda_1}{K_1} - \frac{\lambda_2}{K_2}\right)^2}{\left(\frac{a_1}{K_1}\frac{\lambda_1}{K_2} + \frac{a_2}{K_2}\frac{\lambda_1}{K_1}\right)\left(\frac{a_1}{K_1} + \frac{a_2}{K_2}\right)_2}$$

This shows that as a_1 becomes smaller and smaller the value of A_1 is approximately proportional to $\frac{a_1}{K}$; as K_1 would always have a definite value, this shows that the reduction of the impurities should give a uniform reduction of the absorption.

The first term on the right in Eq. (15) is evidently the final steady value of X_1 as shown by Eq. (11), and the second term shows the variation of X_1 from its initial value at $t = 0$ to its final value as given by Eq. (11). The constant C is determined by the initial boundary condition, and we have

$$X_1 = \frac{E_0}{d}\frac{\lambda_2}{\lambda_1 + \lambda_2} + \frac{E_0}{d}\left(\frac{K_2}{K_1 + K_2} - \frac{\lambda_2}{\lambda_1 + \lambda_2}\right)e^{-\frac{t}{T}} \tag{17}$$

Substituting this value of X_1 in Eq. (5), we have the absorption current in the terms of the applied electromotive force

$$i = \frac{E_0}{d(\lambda_1 + \lambda_2)}\left[\lambda_1\lambda_2 + \frac{(\lambda_1 K_2 - \lambda_2 K_1)^2}{(K_1 + K_2)^2}e^{-\frac{t}{T}}\right] \tag{18}$$

Equation (18) shows that the current starts at a finite value and decreases following the negative exponential relation with the time, as noted experimentally by a number of observers. It approaches a final steady value, fixed by Ohm's law:

$$i = \frac{E_0}{d}\frac{\lambda_1\lambda_2}{(\lambda_1 + \lambda_2)} \tag{19}$$

The exponential term disappears and the current never rises above the value of Eq. (19), if $K_1 : \lambda_1 = K_2 : \lambda_2$. In this case obviously there is no absorption nor residual charge.

The treatment in the original paper is more complete for the building up of a residual voltage. Suppose E_0 is suddenly applied; then

$$Q = \int i\,dt = X_1\lambda_1 dt + \frac{X_1K_1}{4\pi} \tag{20}$$

and in the first instant, since X_1 is finite, the first term on the right is negligible, and

$$E_0 = 4Q\left(\frac{a_1}{K_1} + \frac{a_2}{K_2} + \cdot\ \cdot\ \cdot\right) \tag{21}$$

that is, the instantaneous charge is that corresponding to the geometric capacity, and the initial division of the total electromotive force over all the layers is that corresponding to the series connection of the geometric capacities. If now E_0 is applied for a long time, the unequal conduction currents in the several layers, due to initial electromotive forces determined by the geometric capacities, cause charges to accumulate on the surface of separation until finally an equilibrium condition is reached in which, by Eq. (5), the division of the total electromotive force over the several layers is determined only by their conductivities. The absorption currents of charge and discharge are thus seen to be due to the change from a voltage division due to the different specific inductive capacities, to one due to the different values of resistance, or *vice versa*. The treatment shows by similar methods that after charge the instantaneous discharge is that due to the geometric capacity, and that if then opened, the condenser will build up a residual charge and difference of potential of value:

$$E = E_0\left[\left(\frac{a_1}{R\lambda_1} - \frac{4\pi a_1C}{K_1}\right)e^{-\frac{4\lambda_1}{K_1}t} + \left(\frac{a_2}{R\lambda_2} - \frac{4\pi a_2C}{K_2}\right)e^{-\frac{4\lambda_2}{K_2}t} + \cdot\ \cdot\ \cdot\right] \tag{22}$$

in which R is the total resistance and C the overall geometric capacity. If the ratio of λ to K is the same in all strata, the exponential terms are all equal and the total of the other terms is 0 (in two series); thus $E = 0$ and there is no residual charge. If the ratios $\lambda : K$ are not equal, arrange the terms in the order of decreasing values of K. Then the sum of all the coefficients is evidently 0; hence when $t = 0$, $E = 0$, as required by the condition that t is reckoned from the instant in which the short circuit of the condenser is opened. When t is very great, all the terms disappear and $E = 0$; that is, if left to itself after charge and instantaneous discharge, the condenser builds up a residual electromotive force which rises to a maximum and dies away to zero value. The residual charge leaks away over the conductivities. The instantaneous discharge at any instant t will be EC, C being the geometric capacity, and it has the same sign as the original charge.

Numerical Examples.—It is of interest to examine further the simple case of a two layer condenser, selecting substances whose constants are known. The following example is given by Wagner: The values of K for ordinary dielectrics range approximately from 2 to 10, not a very wide variation. On the other hand, an extremely wide range of values is found for the conductivity. Expressed in (ohms per $\text{cm}^3)^{-1}$, we find such values as the following: ceresin 2×10^{-19}, hard rubber 10^{-18}, quartz 10^{-19}, sulphur 10^{-17}, mica 10^{-15}, paraffin 10^{-16}, glass 10^{-14}, impregnated paper 10^{-11}. There is thus the possibility of a very great difference in the ratio of $\lambda : K$ for even ordinary materials.

Consider, then, unit area of a condenser having a layer of impregnated paper ($K_1 = 2$, $\lambda_1 = 10^{-11}$) and a layer of mica ($K_2 = 8$, $\lambda_2 = 10^{-15}$) each 0.1 cm. thick. The figures for λ_1 and λ_2 are in (ohms per $\text{cm}^3)^{-1}$, *i.e.*, in practical electromagnetic units. Transforming the displacement from electrostatic to practical electromagnetic units of charge, we have

$$D = \frac{K}{4\pi} \cdot \frac{X}{9 \times 10^{11}} = 0.885 \times 10^{-13} KX \frac{\text{coulombs}}{\text{cm}^2}$$

X being in volts per cm.

If a potential difference of E_0 volts be applied, then at the first instant, in accordance with Eq. (9), $X_1d = 0.8E_0$, $X_2d = 0.2E_0$. In the final condition by Eq. (11), $X_1d = 0.0001E_0$, $X_2d = 0.999E_0$. Thus at the first instant, 80 per cent of the total potential difference is over the paper, a very undesirable condition from the standpoint of dielectric strength, while in the final condition owing to the relatively greater conductivity of the paper, practically all of the potential difference is across the mica. The difference in the displacements in the final condition gives the absorbed charge. It is found to be about five times that pertaining to the initial displacement. Thus the capacity as computed from the total absorbed charge is five times as large as the geometric capacity.

The striking values in the above example are due to wide differences in the values of the ratio $K:\lambda$. The values are 2×10^{11} for the impregnated paper and 8×10^{15} for the mica, the latter 4,000 times the former. If the paper be improved by drying so that $\lambda_1 = 10^{-14}$, the ratio $K:\lambda$ for mica is eighty times that for the paper. We have then in the initial state the same values as before, but in the final state $X_1d = 0.091E_0$, and $X_2d = 0.909E_0$. The absorbed charge is reduced by only 2.5 per cent and so the capacity computed from the absorbed charge is still nearly five times the geometric capacity. The difference in the two cases is perhaps most evident in the value of the time constant

$$T = k\frac{K_1 + K_2}{\lambda_1 + \lambda_2}$$

in which k is equal to 0.885×10^{-13}, this constant arising in the transfer of the displacement from the electrostatic to the electromagnetic system. Evaluating T for the two cases cited we find that for the case of mica and undried impregnated paper, the variable part of the current will fall to 1.8 per cent of its final value in 0.35 second; while in the second case of mica and thoroughly dried impregnated paper, the current does not reach 1.8 per cent of its initial value under 354 seconds. It will be noted that owing to the lower conductivity in the second case, a much longer time is required for the absorbed charge to accumulate at the surface of separation and the current dies out much more slowly.

It is well to indicate at this point Maxwell's own idea as to his study of the composite dielectric. He says:

> This investigation shows that a dielectric composed of strata of different kinds may exhibit the phenomena known as electric absorption and residual discharge, although none of the substances of which it is made exhibit these phenomena when alone . . . It by no means follows that every substance which exhibits this phenomenon is so composed, for it may indicate a new kind of electric polarization of which a homogeneous substance may be capable, and this in some cases may perhaps resemble electrochemical polarization much more than dielectric polarization.

It is obvious from the above that Maxwell did not propose the composite dielectric as a complete theory of dielectric absorption. He merely points out that a composite dielectric will manifest the usual anomalous properties of dielectrics. Nevertheless the very fact that the composite dielectric does not invoke any new phenomena, but rests only on properties of dielectrics already well known, has given Maxwell's analysis a wide appeal. As a consequence, it is commonly spoken of as "Maxwell's theory of absorption." This is not surprising in view of the fact that he did not investigate nor definitely propose any other theory.

Experimental evidence in favor of Maxwell's theory is very meager, and chiefly limited to indirect and broadly qualitative confirmatory observations. Cohn and Arons tested the assumption that polarization and conductivity occur independently by means of parallel condensers of different dielectrics and found good agreement. Mixtures of xylol and anilin showed a 10,000-fold variation of resistance with only a one-third variation of dielectric constant. Rowland and Nichols showed that in perfect samples of calcite and possibly quartz, probably the most nearly homogeneous substances available, there is no absorption. Hertz showed that benzine, a homogeneous fluid, when impure shows absorption, which disappears on purification. Arons claims that carefully purified paraffin shows no absorption; this is disputed, however, by Dessau and others. Wagner finds extremely low values of power factor at 5,000 cycles for ceresin and paraffin, but that for a 50 per cent-50 per cent mixture, the power factor was increased several times. Muraoka by careful purification found no absorption in paraffin oil, petroleum, resin oil, turpentine, and xylol. For layers of air and paraffin, absorption appears. Many observers have found that the observed curves of charge and discharge currents, while not obeying the exponential law with a single term, can nevertheless be represented readily by several such terms, as called for in Maxwell's most general case. Its firmest basis, however, is found in the fact that it introduces no new phenomena nor assumptions, but relies only on properties of matter already well known, and on fundamental equations. This, more than any experimental confirmation, accounts for the firm hold that this theory has on the mind of the physicist of today.

The chief disadvantage that the theory suffers is that not only has it had no quantitative nor exact experimental confirmation, but many experiments appear to offer actual contradication. Many observers, of whom particular mention may be made of Curie, Fleming and Dyke, and Trouton and Russ, have found other expressions than the exponential for the absorption current. The phenomenon of conduction in a dielectric may be based on ionic conductivity (Bialobjeske), and so follows a law different from that of metallic conduction. Anderson and Keane have shown that a movement of free electrons in the medium, accumulating at the positive electrode and returning to their equilibrium distribution on discharge, will account for absorption and residual charge. Further, it has been shown that the conductivity of dielectrics may be increased by exposing them to the influence of ionizing radiation. Muraoka found that solutions and certain liquid mixtures show no absorption. Particularly in the behavior of dielectrics under alternating stress, discussed in later paragraphs, noticeable discrepancies have been found by Curtis and others. Comparatively few serious attempts have been made to check Maxwell's theory quantitatively. This is largely due to the very general, if not sketchy, form in which it is developed by its author. In particular, attempts to extend his equations to the alternating case encounter great mathematical difficulties. Pronounced absorption is found in many simple substances which can have in them only relatively small amounts of impurities. While the further removal of these often lowers the absorption, there is some doubt that it ever completely disappears, and it appears remarkable that a combination of such extremely disproportionate amounts of two substances should still function in accordance with the theory. Further, if these small amounts so function, should not greater amounts function in increasing degree? If the amounts of impurities sufficient to account for absorption are too small to control or even observe, then there is little hope that the theory will ever be rigidly confirmed.

Wagner.—Maxwell's development assumes successive layers of different dielectrics, each having different values of λ and K. Many layers must be assumed to account for the results of experiment, and this leads to mathematical difficulties. The charging curve for two substances obeys the negative exponential relation to the time as already noted. Using this relation, K. W. Wagner examines the curves taken on various substances, and states that only a few exponential terms are necessary in any case to account for the curves of experiment, and moreover that the time constants of these terms group themselves more or less closely about a principle value T_0. He is therefore led to attempt a picture of a model dielectric which provides the variation in conductivity and in specific inductive capacity called for by Maxwell, but which avoids the simultaneous occurrence of each in a single dielectric substance. He proposes a pure dielectric medium without conductivity, but with conducting spheres embedded in it, small compared with their distances apart. He investigates this model mathematically at great length; the different time constants of the several terms in the absorption curve are provided by different sets of conducting spheres, mixed with the surrounding dielectric in different proportions. He arrives at equations for the electric displacement, residual charge, time constant, etc., which agree with those of Maxwell's theory in their general form but which involve fewer terms. He also gives an expression for the grouping of these terms involving only two constants of the dielectric medium. He considers the behavior of the model under change of temperature and states that it conforms to the results of observation. The development is highly speculative and pictures a highly specialized interrelation of dielectric and embedded conductor. It seems doubtful whether it could be subjected to satisfactory experimental confirmation. Its only value seems to lie in its offering of a mental picture of a type of structure providing Maxwell's assumptions, while retaining the idea of a homogeneous dielectric free from conductivity.

LECTURE IV

THEORIES OF DIELECTRIC PHENOMENA *(Continued)*

THEORIES OF ABSORPTION BASED ON MOLECULAR STRUCTURE

Even the simplest natural phenomena leave us groping when we seek their ultimate understanding as a sequence of cause and effect. If we pursue our effort to understand the nature of each cause, we soon come to the limit of our exact knowledge and are led into the realm of metaphysical speculation. For the most part, in the field of physical phenomena, we are usually content if we can express them in terms of the fundamental mechanical relations of force, mass, motion, and energy. In our human make-up we are conscious of the effects of these relations, we react to them each hour of the day, and we have set up units and methods for their measurement. Consequently they are accepted as foundation stones upon which we can erect the structures of theories accounting for more complex natural phenomena. Thus the tendency and effort of modern physics is to explain all natural phenomena in terms of molecular, atomic, and subatomic masses and motion. In the special fields of electricity and magnetism it is only by means of their manifestations through the laws of mechanics that we are able at all to bring within the science and art and engineering application these most mysterious of all natural phenomena save only that of life itself. Explanations and theories are thus relative terms only, and we are usually satisfied to consider that we have a theory of any new natural phenomenon, if we can explain it in terms of more fundamental phenomena whose laws are well recognized, even though they may not themselves be completely understood. Thus, since the earliest discoveries of the properties of dielectrics, many explanations of their origin and nature have been suggested. Few if any of these suggestions, however, have risen to the dignity of a theory substantiated by quantitative experimental test.

Bearing these things in mind, we see that Maxwell's theory of dielectric absorption, briefly reviewed in the foregoing lecture, is truly a theory in the best sense of the term. The fact that its author admitted the possibility of other causes leading to the same results still leaves it as a theory. It explains a complex phenomenon in terms of simpler phenomena whose laws are known. Its whole structure is erected on the well-known properties, electric conductivity and specific inductive capacity. The fact that the ultimate nature of these two properties is not understood constitutes no difficulty. These two properties are so well known and have been so widely studied that they are recognized not only as fundamental in the entire field of electrical science, but they also enter into both the language and the laws describing all electrical phenomena. The strong appeal of Maxwell's theory consists in the fact that it is based solely on these simple and well-recognized properties and does not invoke any others.

Other theories of absorption of the remaining three classes differ essentially from that of Maxwell by reason of the fact that for the most part they are dependent upon speculation as to molecular and atomic phenomena, which are not yet clearly recognized and which in the theories are usually introduced through special forms of mathematical functions. In many cases the results of these theories will be found to agree quite well with the results of experiment. We shall see, however, that these agreements often resolve themselves into ingenious efforts to find mathematical explanations for experimental results which are already known. Nevertheless, these theories have very great value. Maxwell's theory has not been proved correct by quantitative methods. Moreover, it appears quite probable that there may be present in the behavior of many anomalous dielectrics other phenomena than provided by Maxwell's theory. In many of the theories about to be described, there is ample suggestion of the nature of some of these extraneous but also important manifestations of dielectric behavior.

As we have already stated, the great appeal made by Maxwell's theory of absorption is that it is based only on electrical phenomena already known and well understood. It introduces no new hypothesis as to the electrical behavior of matter. Specific inductive capacity and conductivity account for all. It is natural that the attempt should have been made on this simple basis, for in Maxwell's time little was known of the molecule; the electron had not been discovered, and information as to the structure of the atom was still far in the future.

In spite of the appeal of its simple elementary assumptions, however, Maxwell's theory did not satisfy contemporary physicists and engineers. This was largely due to the incomplete form in which it was developed and to the difficulties found in attempts to verify it experimentally. No substance, however simple or pure, could be found which was entirely free of absorption. It was necessary to assume that impurities in amounts too small to be removed or even discovered were nevertheless sufficient to cause noticeable absorption effects. Moreover, many of these phenomena as observed in experiment were quite suggestive of the behavor of viscous and elastic materials under mechanical stress. It was therefore natural that other theories of absorption should be proposed and that they should have taken the forms of analogies with other elastic and viscous phenomena. Maxwell himself pictured an elastic ether modified by molecules suspended in it. Thus for a reasonable mental picture it was only necessary to extend Maxwell's idea so as to include the property of viscosity. In fact Maxwell suggested to Hopkinson that the latter should investigate the question of residual charge in a manner similar to that used by Boltzman for the after effects of mechanical strain.

Group 2

Hopkinson.—John Hopkinson was one of the earliest systematic students of dielectric absorption. He was a student of both Sir William Thompson and Maxwell and had the benefit of their suggestion and advice in his experiments in this field. His work is characterized by ingenuity and accuracy, and his four papers on this subject should be read by every student of dielectric phenomena. His first paper, delivered in 1876, reports a number of experiments on the decay of the residual charge of a condenser with particular reference to the duration and polarity of the charging potentials during its previous history. He shows a number of such discharge curves, two of which we reproduce here (Fig. 17), showing reversals of polarity and indicating their relationship to preceding periods of charge. He proposes as explanation that the dielectric consists of a mixture and that the components have different rates of approaching complete polarization and decay. We have here a suggestion of Maxwell's theory, and we may imagine Hopkinson's proposal as reflecting his conversation with Maxwell. We have already noted that Hopkinson also quotes Sir William Thompson as follows: "The charges go out of the glass in the inverse order to that in which they come in." In this paper he also attempts to find an equation expressing the rate of decay of residual charge and concludes that his curves cannot be expressed by a negative exponential term or even by two of them, but that several such terms might be sufficient.

It is in his second paper in 1877 that Hopkinson, inspired by Maxwell, developed the Boltzman theory for an elastic residual phenomena. He develops an equation for the residual potential of a dielectric, analogous to the equation for the angular twist of a torsion thread under a succession of couples of various magnitudes and direction. Each couple applied for some time is supposed to cause an initial, and a sustained, gradual yield with time, that is, an increase of angular displacement, and the decay of this is reflected in subsequent twisting couples. Hopkinson's formula is substantially as follows:

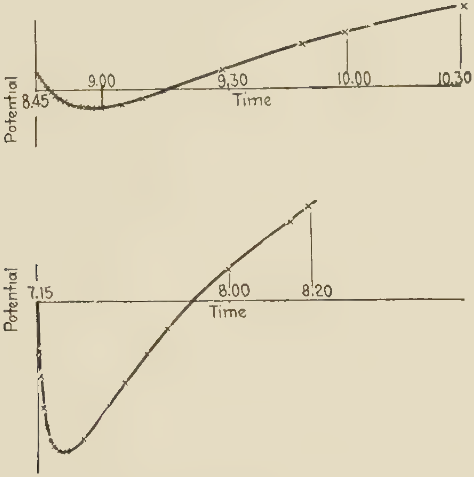


FIG. 17.—Reversal of residual charge. (*Hopkinson.*)

$$D(t) = \frac{K}{4\pi}E(t) + \int_0^{\infty} E(t-u)\cdot\varphi(u)\cdot du \qquad (1)$$

This formula states that the actual displacement $D(t)$ in a dielectric at any instant t is made up of two terms. The first is the normal displacement as determined by the dielectric constant of the medium and by the instantaneous value of the voltage $E(t)$

in accordance with fundamental theory. The second term is the sum of the residual effects of all foregoing values of E considered as a function of an elapsed time u. Equation (1), therefore, has the important property of showing the variation of the displacement when the electric field strength has varied in the past in accordance with any given function of the time $E(t)$. Every such residual effect of foregoing values of E decreases with the elapsed time u in accordance with a definite function $\varphi(t)$, and the single residual effects are superimposed on each other. The formula is in fact a statement of the principle of superposition, which therefore becomes a fundamental assumption of Hopkinson's elastic theory. He concludes from many experiments that his equations are only roughly approximated, but they are quite sufficient to support his principal conclusion that the effects on a dielectric of past and present electromotive forces are superposable. Hopkinson also made one of the first efforts to extend the principle of superposition to the case of alternating electromotive forces. The experiments led him into studies of the absorption corresponding to extremely short intervals of time, for which he developed an ingenious pendulum method which enabled him to trace the curves of absorbed charge at intervals of 0.00002 second over the initial periods of charge and discharge.

Houllevigue.—M. L. Houllevigue of the University of Lyons has given us one of the simplest and best developments of an elastic theory of absorption. He is struck by the fact that Maxwell's theory accounts neither for the general form of the curve of discharge of the condenser, nor for the departures from the relation between the specific inductive capacity and the index of refraction ($K = n^2$), which follow from Maxwell's general electromagnetic theory. Houllevigue extends Maxwell's idea of a dielectric and proposes that the molecules of the dielectric are suspended in the ether. The ether is attached to each of the molecules by some form of elastic connection. Under the electric field the ether is displaced in the direction of the field until equilibrium with the elastic reaction is reached. This is an instantaneous effect in the case of a condenser with perfect dielectrics. In the case of a dielectric with absorption, the assumption is made that the molecules yield under the tension, their mean positions are displaced in the direction of the field, and the displacement of the ether is increased by the amount of this molecular displacement. Thus the phenomenon of absorption is supposed to have its origin in the viscosity of the material of the dielectric. The total displacement in an imperfect dielectric thus consists of two parts: (*a*) an instantaneous displacement of the ether until the elastic forces between ether and molecules are in equilibrium, and (*b*) an additional gradual yielding of the molecules under the tension caused by the first displacement. This point of view introduces the laws of viscosity in which the forces are determined, and displacement depends, not only on the displacement itself, but on the rapidity with which it takes place; that is, there is a frictional reaction proportional to the velocity of the displacement. Such friction would naturally manifest itself as heat, of which we have conspicuous evidence in the heating of condensers under alternating stress.

These simple ideas lead to a correspondingly simple mathematical treatment which Houllevigue extends to the alternating case. Such a theory, based as it is on simple mechanical assumptions, makes a strong appeal. Moreover, so far as concerns the behavior of a dielectric under continuous potential, it would be found perhaps to satisfy the results of experiment as well as any other theory. F. W. Grover has examined Houllevigue's expressions or the alternating case, however, and compared them with his own observations on a number of condensers. He finds that for increasing frequency this theory calls for a minimum value of the apparent capacity smaller than the geometric capacity, and increasing thereafter. Many observers have noted a uniform decrease of the apparent capacity with increasing frequency approaching the value of the geometric capacity at extremely high frequencies. These results, therefore, constitute a serious objection to such elastic theories as that of Houllevigue.

Pellat.—Another important theory is that of M. H. Pellat. He makes no attempt at an explanation of underlying causes of absorption, nor of the ultimate nature of the processes involved. He proves experimentally that dielectrics subjected to electric force show an initial instantaneous polarization, to which is added an additional polarization which changes with the time. This second variable component polarization leads to the usual residual phenomena. He therefore proposes to account for these phenomena qualitatively and quantitatively by making an assumption as to the law followed by the variation of the polarization in time. The law he proposes is that the second or variable part of the polarization tends to a final value which is proportional to the applied electric force, and that its rate of change is always proportional to its difference from its final value.

Expressed then in terms of the fundamental theory, Pellat considers the displacement as divided into two parts

$$D(t) = D'(t) + D''(t) \tag{2}$$

The first of these is

$$D'(t) = \frac{KE(t)}{4\pi} \tag{3}$$

in which E is the electric intensity.

The second part as described above is assumed to obey the equation

$$\begin{aligned} \frac{dD''(t)}{dt} &= \alpha[D''(\infty) - D''(t)] \\ &= \alpha\left[\eta\frac{K}{4\pi}E(t) - D''(t)\right. \end{aligned} \tag{4}$$

Pellat calls D' the "fictitious" and D'' the "true" polarization. In this he apparently assumes that the usual instantaneous displacement is less a property of the substance than of the ether, while the second part is essentially a property of the dielectric. Others have preferred the terms "normal" and "viscous" components of the total displacement.

In accordance with the foregoing, we have for *constant* electric intensity E_0, and for $t > 0$ we have

$$D''(t) = \eta\frac{KE_0}{4\pi}(1 - e^{-\alpha t}) \tag{5}$$

$$i_1^1(t) = \frac{dD''(t)}{dt} = \alpha\eta\frac{K}{4\pi}E_0e^{-\alpha t} \tag{6}$$

Thus the function showing the time variation of the reversible anomalous current takes the simple negative exponential form. Integrating the above Eq. (6) we have, for any type of variation of the electric intensity $E(u)$ in the past, the variable displacement at any instant t:

$$D''(t) = e^{-\alpha t}\int_0^t \alpha\eta\frac{K}{4\pi}E(u)e^{-\alpha u}\,.du$$

$$D''(t) = \alpha\eta\frac{K}{4\pi}\int_0^\infty E(t - \omega)e^{-\alpha\omega}\,.d\omega \tag{7}$$

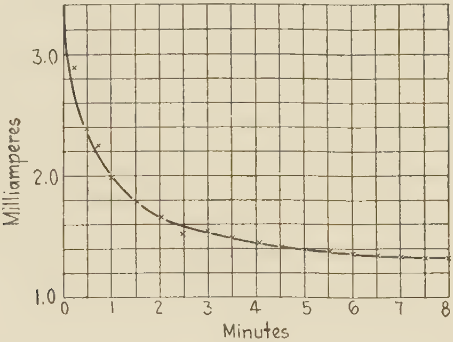
in which $u = t - \omega$. The total displacement is

$$D(t) = \frac{K}{4\pi}E(t) + \alpha\eta\frac{K}{4\pi}\int_0^\infty E(t - \omega)e^{-\alpha\omega}\,.d\omega \tag{8}$$

Thus the variation of the displacement satisfies the principle of superposition and the theory of Pellat is seen to be a special case of that of Hopkinson in which $\varphi(t)$ is of the form $Ae^{-\alpha t}$.

The simplicity of Pellat's assumption as to the variation of the displacement, and the close approximation to observations in his expression for the anomalous charging current, make a strong appeal, in spite of the absence of all suggestion of underlying explanation. In order to supply this deficiency, von Schweidler has extended Pellat's proposal in considerable elaboration, with the aim first to bring it more nearly into accord with observation, and second to present a picture of underlying mechanism.

von Schweidler.—Pellat's theory results in the simple negative exponential variation of the reversible anomalous current with time. This relation also results from Maxwell's theory for the case of a mixture of two substances. We have already noted that curves taken from experiment are rarely in accord with this simple relation, nor with any of the other relations as given in our second lecture. A common method for identifying the results of experiment with theory is to assume the presence of other dielectrics in small amount, that is, to resolve the experimental curve into a number of other curves, each one obeying the simple exponential or other form of continually decreasing relation. Hopkinson, for example, tried the negative exponential relation to some of his curves and concluded that more than two terms were necessary to represent his results. We have already noted that Wagner applied this method in his discussion of Maxwell's theory. Steinmetz has shown that the absorption-current curve



$$i = 1.20 + 0.681\epsilon^{-0.00420t} + 1.39\epsilon^{-0.0268t}$$

FIG. 18.—Charging current of a three-phase cable. (*Steinmetz.*)

of a certain high voltage cable was very closely represented by three exponential curves (see Fig. 18). This analysis of Steinmetz is very suggestive. Impregnated paper insulation is very complex in character and always shows a high degree of absorption. That its curve should be susceptible of such simple analysis is an indication perhaps that so far as insulation is concerned, the exact form of fundamental absorption curve is of relatively small importance.

E. von Schweidler has adopted this method in an extension of the theory of Pellat. He assumes that the anomalous displacement D'' consists of a number of turns, each one of which satisfies the Pellat differential equation, but each having different values of η and α. Thus the expression for the anomalous displacement consists of a series of terms. The dielectric is characterized by the normal constants K and λ, and in addition by a so-called distribution function which defines the mode of occurrence of the constants η and α. In order to give the theory a physical interpretation he traces the cause of absorption to high-frequency oscillations of molecular charges. Noting that these oscillations are involved in the phenomena of the dispersion and absorption of electromagnetic waves, he assumes that there are also other types of molecules present. The oscillation of the charges in these molecules is aperiodically damped so that even in electric fields which change relatively slowly, these charges return to their equilibrium positions with a noticeable time lag. Moreover, it is necessary to assume that there are as many kinds of molecules as are required by the distribution function. In order to simplify the determination of the distribution function, he proposes a method used by Wiechert in a theory of elastic "after working," and which we have already mentioned in describing Wagner's extension of Maxwell's theory. By this expedient the various sets of constants group themselves about a single predominating set, and the number of sets and their proximity to the central set is indicated by a single constant of the material. von Schweidler states that in general only three sets of constants, *i.e.*, three terms of the Pellat type, are sufficient for a calculation of all measured values both continuous and alternating.

As regards experimental confirmation, Grover, from studies on paper condensers, concluded that, of several theories examined, the Pellat theory as modified by von Schweidler was the only one that could be made to give quantitative results in agreement with the observations. The quantities studied were changes in capacity and phase difference, with frequency and temperature. It is to be noted, however, that Grover did not examine Maxwell's theory as extended by Wagner, which appeared later, and which involves the same type of variation of the anomalous charging current, and an entirely analogous method of assuming a number of terms and of studying their grouping. It appears certain that an equally good agreement would have been obtained from Wagner's equations. In fact, it is safe to say the same of any theory providing for the medium a sufficient number of terms, all obeying a continually decreasing function $\varphi(t)$ of relatively simple form, but with different values of the constant terms. It appears not improbable that Wagner, not caring for von Schweidler's idea of slow-period molecular oscillations, set out to picture a structure of an anomalous dielectric which would involve only the fundamental properties of specific inductive capacity and conductivity, thus adhering to Maxwell's ideas.

It is proper to say that the extension of Maxwell's theory by Wagner and that of Pellat's theory by von Schweidler really adds nothing to our knowledge. Their analyses are highly artificial and merely offer us mental pictures of a dielectric structure which might account for the experimental facts. Wagner must show us a dielectric which possesses only specific inductive capacity and electric conductivity in some portions and discrete conducting particles in others, and von Schweidler must show independent evidence of the existence of his various kinds of molecules before their respective so-called theories can be accepted as such. In fact, much the same may be said of all of the theories which have been discussed in this lecture. They offer us mental pictures of structures which might account for the facts, but present no certain evidence. The only certain conclusions which seem to be reached in each case are that they are all in accord with Eq. (1), *i.e.*, that they obey the principle of superposition, that equations may be derived for mixtures of several variations of the same dielectric, all obeying Eq. (1), and that at best it is generally necessary to assume the presence of two or more disturbing materials, or varieties of the same fundamental material.

Dielectric Hysteresis.—The common phenomena of absorption which we have been discussing are frequently referred to as arising in some form of dielectric hysteresis. Hysteresis in iron is assumed to be a molecular phenomenon, and so dielectric hysteresis as a theory finds place in this lecture devoted to those theories which attempt to explain dielectric absorption in terms of molecular behavior.

There is at first sight much evidence in favor of a theory of dielectric hysteresis. Experimental curves taken between electric intensity and intensity of polarization and residual charge have frequently been reported in the form of closed loops. As an example, Fig. 19 shows a series of such curves reported by M. Mouline, plotted between intensity of polarization and electric force. The similarity to the hysteresis loops of magnetic material is obvious. If the familiar residual phenomena, and the losses in dielectrics under alternating stress, are due to a viscosity of the dielectric medium, then there is good reason for the use of the term "hysteresis." But this is by no means certain. It is my opinion that there is not sufficient evidence in the phenomena of absorption, as we observe them, to warrant the use of the term "hysteresis" in describing them. Moreover, there are certain definite objections to the use of the word in this connection.

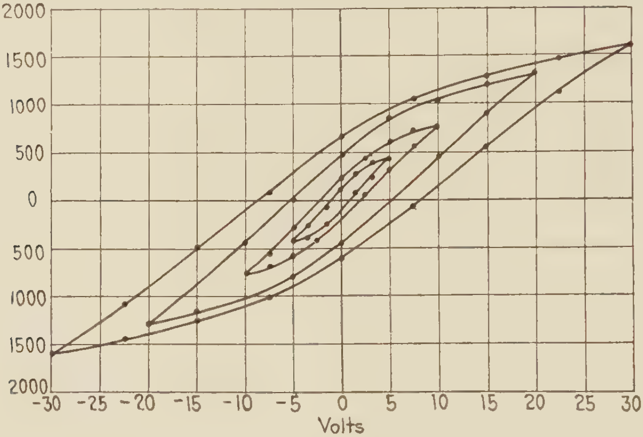


Fig. 19.—Dielectric hysteresis loops. (*Mouline.*)

The term "hysteresis" apparently has never been clearly defined. If it be assumed to apply generally to any phenomenon in which the value of the effect of a force depends not only on the value of the force but on the preceding history of the system, then there are certain grounds for using the term to describe anomalous dielectric behavior; for this property is possessed by magnetic, dielectric, and elastic media. The term as applying to dielectrics apparently was used first by Steinmetz shortly after his famous work on magnetic hysteresis. It has been so generally used ever since as to make it evident that there is wide feeling that the two phenomena are similar in underlying character. The grounds for this are probably (*a*) the close parallel between the fundamental theories of magnetic and pure dielectric phenomena (μ and K); (*b*) the cyclic electrification of anomalous dielectrics, often showing the relation between E, the electric force, and Q, the total charge, as a closed curve; and (*c*) the dielectric loss per cycle as usually proportional to E^n, n in certain cases having been reported as low as 1.6.

But if the word is understood to describe the complete phenomenon as it occurs in magnetic material, then it should not be used as applying to dielectrics, for the behavior of the latter differs in important particulars from that of the former. The work of Hess, Beaulard, Germanischskaja, and others has shown the following essential differences in dielectrics as compared with magnetic materials: (*a*) There is no saturation region, D being proportional to E, up to breakdown. (*b*) The stationary final value of the displacement is always proportional to the field strength, for both ascending and descending values although a time element is involved. (*c*) The dielectric loss per cycle varies widely with the frequency, while the magnetic loss does not, except in the extreme upper range. (*d*) There is no dielectric permanent residual displacement nor coercive force. (*e*) The anomalous charging current may not be explained on the basis of hysteresis. Hess discusses the difference between hysteresis and viscosity and considers that the phenomena in dielectrics have the properties of the latter rather than of the former. In hysteresis there are two values of B for every value of H, dependent only on the direction from which H is approached and on $H_{\max}$. They are independent of the rapidity of the cycle of H within a wide range. In a viscous medium, however, there can be any number of B's for the same H, dependent on the rapidity of the cycle of H, but for very slow cycles there is only one value of B for each value of H. Thus hysteresis is clearly evident under slow cycles in iron, but not in dielectrics.

LECTURE V

THEORIES OF DIELECTRIC PHENOMENA (*Continued*)

THEORIES OF ABSORPTION BASED ON ATOMIC STRUCTURE AND ON ANOMALOUS CONDUCTIVITY

Our third group of theories is that in which effort is made to explain the anomalous behavior of dielectrics in terms of the motion of electrons within the atom. The idea that electric displacement is due to the movement of charges within the molecule or atom is an old one, dating back to Helmholz (1870), and has frequently been invoked in a general way to explain anomalous displacements. For example, we have seen that von Schweidler in his extension of Maxwell's theory has assumed that charges within the molecules oscillate with different degrees of damping. Moreover, the notion of dielectric hysteresis with some writers involves the conception of the motion of charges within the molecule. In many instances the idea of the motion of charges is not limited to the molecule or atom, but is extended to much longer paths through the body of the dielectric. Many of the phenomena in liquid dielectrics, for example, as we shall see below, are almost certainly due to the motion of molecular charges and aggregates through considerable distances. This idea has also recently been extended to the case of solid dielectrics. Anderson and Keane have proposed that a slow drift of electrons through the mass of the dielectric to the electrodes leads to an explanation of the anomalous charge and discharge currents. These, however, are little more than suggestions as compared with the single outstanding theory of this class, namely, that of Décombe.

Group 3

Décombe, in seeking an explanation of the heating of condensers, notes the remakable and very general fact that most thermodynamic modifications are inseparable from noticeable electric phenomena. Thus, mechanical deformations are always accompanied by both heat and electrification (triboelectric, and piezoelectric effects); capillary deformations, shock, cleavage, etc., all result in both heat and electric manifestations. Similar effects are noticeable in chemical relations, as in the thermal variations of crystals, the thermoelectric cell, the Thomson and Peltier effects, etc. He thus concludes that dielectric absorption and losses are also to be explained in terms of motions or deformations of electrons within the atom, and he bases an interesting and extended development of this theory on the general electron theory of Lorentz:

Separating the displacement of the ether from that of the material as in Maxwell's theory, we have

$$K = 1 + 4\pi k \tag{1}$$

so that we have Maxwell's displacement

$$D = \frac{KE}{4\pi} = \frac{E}{4\pi} + Ek \tag{2}$$

the term Ek is the intensity of polarization I in the dielectric. Décombe writes $m = Ek$ to indicate his adoption of the idea that the polarization of the dielectric is equivalent to or identical with Lorentz' displacement of the electron within the atom.

For a unit condenser Eq. (2) may, therefore, be written

$$q = k_0E + m \tag{3}$$

in which $k_0 = \frac{1}{4\pi}$. Further, according to Lorentz, we have

$$a\frac{d^2m}{dt^2} = E - bm - c\frac{dm}{dt} \tag{4}$$

This is the familiar equation of mechanical equilibrium between an applied force and forces of reaction due to acceleration, elasticity, and friction attendant upon the displacement m. Here E is the electric intensity, and a, b, and c are constants of electrical inertia, elasticity, and friction associated with the displacement or movement of the electron within the atom. For frequencies less than those corresponding to light, the electron may be assumed to have no inertia, and so the acceleration term disappears, and we have

$$E = bm + c\frac{dm}{dt} \tag{5}$$

Equations (3) and (5) are the fundamental equations of Décombe's treatment. He tests them with two experimental facts, namely, the principle of superposition, and oscillograms and power losses as observed by Hochstaedter on high-voltage cables under alternating stress.

He states that Eq. (5) is consistent with the principle of superposition as follows: If m_1 is a general solution of

$$E = bm + c\frac{dm}{dt} \tag{6}$$

and m_2 a general solution of

$$E' = bm + c\frac{dm}{dt} \tag{7}$$

then

$$E + E' = bm + c\frac{dm}{dt} \tag{8}$$

is satisfied by $m_1 + m_2$.

Décombe apparently assumes that this is a sufficient proof that Eq. (5) is consistent with the principle of superposition. As a matter of fact, it proves that only in the particular case when E_1 and E_2 act together the resulting behavior of the polarization during the application of E_1 and E_2 will be the joint effect of the polarizations due to E_1 and E_2 acting singly. As has been clearly pointed out by Bouasse, however, this says nothing as to the period following the removal of the E's. In this case Eq. (5) calls for the same law of return (une exponentielle) to the neutral condition in all cases. Experience shows, however, that the decay of polarization following removal of applied voltage may follow a variety of types of variation, sometimes reversing, and depending only on its previous history. This contradiction alone appears sufficient to cast much doubt on the accuracy of Décombe's assumptions.

We may note, however, that for E constant, that is, for continuously applied constant value of electromotive force, Eq. (5) may be solved for m, and so $\frac{dm}{dt}$ may be evaluated. Now $\frac{dm}{dt}$ as the time rate of change of the polarization is the anomalous charging current. The value so obtained by the solution of Eq. (5) shows that the anomalous charging current varies in a negative exponential relation to the time, as also results from Maxwell's theory, and as frequently observed in experiment.

Décombe was attracted to this problem by the losses in dielectrics under alternating stress and as manifested in the so-called Siemen's heating of condensers. His development therefore extends to the alternating case. He finds in the measurements on high voltage cables taken by Hochstadter a means for comparing his theory with the results of experiment. Equations (3) and (5) give us the fundamental assumptions of his theory.

By means of the oscillograph Hochstadter was able to take simultaneous curves of the time variation of the difference of potential E applied to the cable, and of the strength of the charging current. From this by graphic integration he was able to obtain the corresponding curve of the time rate of change of the total charge, and also that representing the energy dissipated during charge. Décombe studied these curves and shows that if one multiplies all the ordinates of the curve of E by the same coefficient C (equal to the ratio q_m to E_m, *i.e.*, to the static capacity of the condenser), the curve obtained may be superposed exactly on that of q by a slight advance in phase. If then E varying with the time is expressed as $E = F(\omega t)$ (where $\omega = 2\pi f$), the instantaneous value of the total charge may be expressed in the form

$$q = CF(\omega t - \psi) \tag{9}$$

where ψ is a very small angle. Expanding and equating to q as given in Eq. (3), the polarization is found to have the same form:

$$m = kF(\omega t - \Phi) \tag{10}$$

in which k is the dielectric susceptibility and

$$\Phi = \frac{4\pi C\Psi}{C - 1} \tag{11}$$

Equation (10) thus shows that the curve of variation of m, the polarization, is of the same form as that of q, the charge. It is an essential feature of Décombe's theory that the anomalous properties of dielectrics find their origin in the behavior of the polarization. As a consequence, he prefers to consider k rather than C a fundamental constant of the condenser. In fact, he states that C, the electrostatic capacity, being made up in two parts, one within the ether and the other in the material, has only an apparent constancy. This variation is rarely noted owing to the small value of the phase difference between q and E. Using m and k, therefore, fundamental Eq. (3) becomes

$$q = k_0F(\omega t) + kF(\omega t - \phi) \tag{12}$$

which states that the polarization of the dielectric lags behind that of the ether and so behind the electric field. If now the periodic function $E = F(\omega t)$, and also Eq. (10) be developed in Fourier's series, and terms of higher order in Φ be neglected, on substitution of the values of E and m in (Eq. (5), it is found that this equation is identically satisfed with the following values for b and c.

$$b = \frac{1}{k}; \tag{13}$$

$$c = \frac{\phi}{k\omega}; \tag{14}$$

That is to say, that in the case of an alternating electric field, fundamental Eq. (5) is verified if one replaces m by its value as found experimentally from equations such as Hochstaedter's. The constants of the equation are given in terms of quantities which may be measured. The constant of elasticity b is seen to be inversely proportional to k, while the frictional or energy-consuming constant c is directly proportional to the phase difference ϕ, and inversely proportional to the frequency and to the susceptibility k. Décombe also shows that this is a general property which obtains for a field varying in any way with the time.

Having shown that these fundamental equations are consistent with the experimental facts observed under continuous potentials, and with the behavior of the charging current under alternating potentials, Décombe seeks for other experimental confirmation. He therefore considers Maxwell's theory and the work of Hess and Pellat, and finally he extends his equations to the explanation of the heating of dielectrics under alternating stress and compares the indications of this theory with the result of experiment.

In considering Maxwell's theory he finds at once the same difficulty found by others, namely, that the form in which the theory is presented by its author offers no simple method for experimental confirmation. As a consequence Décombe finds no experiments bearing directly on Maxwell's theory. He therefore turns to the work of Hess, who proposed as an equivalent of Maxwell's layer dielectric a single layer of homogeneous dielectric substance in which there are embedded small particles of conducting substance. Hess shows that such a dielectric will show the ordinary effects of residual charge and, moreover, that its behavior will be the same as that of a system of two condensers in series, one of which is shunted by a small resistance. Décombe states that if the heterogeneous layer of Hess, or its equivalent system of condensers, be placed in an alternating electric field, the instantaneous charge q of the system satisfies a differential equation of exactly the same form as that obtained by eliminating m between the two fundamental Eqs. (3) and (5). He therefore emphasizes the fact that his equations are not only of a form sufficient to account for the phenomena of residual charge, but they have the advantage over those of Maxwell and Hess in that they do not require any special assumption as to the constitution of the dielectric. Dielectric substances differ as regard residual effect because their atoms have different internal structures. It is not necessary to assume a mixture of two dielectrics to account for residual charge. Décombe also points out that according to Maxwell's theory the losses in dielectrics are due to their conductivity alone, whereas numerous experiments have shown that such losses are far in excess of those which would be accounted for by conduction. He suggests in fact that both Maxwell's layer dielectric and Hess' equivalent system of condensers are in fact only particular models or combinations which behave in a manner similar to that in a real dielectric. He considers that the true dielectric provides in its polarization m and in the friction term $c\frac{dm}{dt}$ all that is necessary for accounting for the phenomena of residual charge and loss. He also calls attention to the fact that the term $c\frac{dm}{dt}$ in the equation of Lorentz has a true experimental origin. It is necessary to account for the existence of the well-known phenomena of luminous absorption. According to Décombe, therefore, luminous absorption and dielectric absorption are both due to frictional resistance encountered by the motion of electrons inside the atom.

It will be recalled that, in Pellat's extension of Hopkinson's theory, he assumes that the current of polarization at any instant is proportional to the excess of the final polarization M over the instantaneous polarization m. Moreover, it was shown that this assumption leads to results consistent with the principle of superposition. Décombe shows that his fundamental Eqs. (3) and (5) lead to an expression which is consistent with Pellet's assumption. The reasoning is as follows: Let a constant difference of potential E be applied to our elementary condenser. After a sufficient time the polarization reaches its final steady value M. Thus $\frac{dm}{dt} = 0$ and by Eq. (5) $E = bM$. Substituting these relations in Eq. (5), we have

$$\frac{dm}{dt} = \frac{b}{c}(M - m) \tag{15}$$

which is obviously equivalent to a statement of Pellat's assumption. Décombe recalls that Pellat has shown that his assumption is in accord with experimental observations of J. Curie, and he thus finds additional support for his own theory.

It is evident, however, that Pellat is chiefly interested in the losses in dielectrics and the heating of condensers (Siemen's heat) under alternating stress, and the explanation of these losses offered by his theory. If one subjects an ordinary elementary parallel plate condenser to an alternating difference of potential $E = f(\omega t)$, the element of absorbed energy is given by Edq. Now in accordance with Eq. (3) dq is of the form $dm + k_0dE$, and consequently the element of absorbed energy is of the form

$$dW = Edq = E(dm + k_0dE) \tag{16}$$

Integrating this expression from zero to T, T being the time duration of one period, we obtain for the energy W dissipated in one period,

$$W = \int_0^T Edm \tag{17}$$

If we replace in the value for W the quantity E by its value as derived from fundamental Eq. (5), we obtain, noting that the integral of Ede and mdm each through a closed cycle is equal to zero

$$W = \int_0^T c\left(\frac{dm}{dt}\right)^2 dt \tag{18}$$

Equation (18), which gives the energy dissipated in one period, shows us that the element of absorbed energy is proportional to the square of the polarization current; that is to say, Décombe finds that the heating and loss in dielectrics is attributable to the polarization of the volume of the dielectric, that is to say, to that portion of the displacement which has its origin in the dielectric. Furthermore, the magnitude of the loss is proportional to the square of the current of the polarization.

Using Eq. (11), expanding the periodic function in accordance with Fourier's series, deriving the value of $\frac{dm}{dt}$, substituting in Eq. (18), and evaluating the latter, Décombe finds

$$W = cAk^2 \tag{19}$$

in which A is a constant. Recalling that c is inversely proportional to ω, we see that the energy W dissipated in one period is independent of the frequency. Further replacing c by its value, we find

$$W = Ak\Phi \tag{20}$$

which shows that the energy dissipated per period is proportional to the angular displacement Φ of the polarization behind the electromotive force, if the value of Φ is not too large. Décombe points to the results of Steinmetz and Hochstaedter as confirming the laws indicated by Eqs. (19) and (20).

Summarizing the point of view of Décombe, we see that heating and, in fact, the anomalous properties in general of dielectrics are caused by the term $c\frac{dm}{dt}$ of Eq. (5). In the absence of this term the anomalies of the dielectric disappear. From the point of view of electron theory, polarization results from the deformation of atoms, and the current of polarization $\frac{dm}{dt}$ must be considered proportional to the rapidity of deformation. It follows, therefore, that the anomalous properties of dielectrics should vanish in all cases in which the atom is not deformed, or in which the rapidity of deformation is infinitely small. This is precisely the character of the phenomenon of viscosity, and Décombe's final conclusion is that we must attribute the residual phenomena of dielectrics, and consequently the heating of dielectrics, to a viscosity arising within the atom.

The theory of Décombe is very striking in its simplicity and in its general agreement with the results of observation. It is strengthened by the firm basis now obtained by the electron theory. It leaves something to be desired, however, in its explanation of residual charge, and of the principle of superposition, and in its agreement with the results of experiment, notably in the matter of loss under alternating potential difference. It may be said of this theory, and of those in the foregoing class, that they do not account for the pronounced effect on both dielectric absorption and loss of very small quantities of moisture and impurities. This theory calls for a loss increasing directly as the alternating frequency. There is a considerable variance on this question in the results of different observers. Some have found a linear relation between frequency and loss; others a loss increasing as a fractional power of the frequency; and still others indicate a loss independent of the frequency. In its simplicity and in view of the independent knowledge now available of electron motion within the atom, the theory makes a powerful appeal, but a more direct experimental confirmation appears necessary before it can receive serious consideration.

A. Anderson and T. Keane have offered an interesting explanation of the phenomenon of residual charge. It should be mentioned in a review of theories of the third class, although it has not been developed so as to apply to other anomalous dielectric properties.

If, instead of Maxwell's layer dielectric, we assume a dielectric in which the resistivity and specific inductive capacity vary *continuously* from one parallel plate electrode to the other, and if we consider the final steady state of such a condenser under a continuous difference of potential, we have from the equation of continuity of current flow and the equation of Poisson

$$\frac{d}{dx}\left(\frac{1}{\lambda}\frac{dV}{dx}\right) = 0 \quad \text{and} \quad \frac{d}{dx}\left(K\frac{dV}{dx}\right) = -4\pi\rho \tag{21}$$

in which λ, K, and ρ are the specific resistivity, the specific inductive capacity, and the volume density of charge, respectively. Hence, if c is the current per unit area

$$C = -\frac{1}{\lambda}\frac{dV}{dx} = \text{constant}$$

$$\therefore \quad C\frac{d}{dx}(K\lambda) = 4\pi\rho \tag{22}$$

and by integration the whole quantity of electricity stored in the dielectric per unit area of plates is

$$\frac{c}{4\pi}(K_0\lambda_0 - K_1\lambda_1) \tag{23}$$

where K_0, λ_0 and K_1, λ_1 are values at the two surfaces of the dielectric. Thus the total residual charge depends only on the constants of the dielectric at the surfaces. Furthermore, as the residual charge is always of the same sign as the primary charge, it follows that $K_0\lambda_0$ is always greater than $K_1\lambda_1$ when a positive current goes from the first plate to the second, that is, when V is positive. When V is negative, $K_0\lambda_0$ is always less than $K_1\lambda_1$. We have then the interesting conclusions that the surface values of one or both the quantities K and λ depend on the field of force. This leads the authors to their suggestion that the resistance of the dielectric is dependent on the electron density in accordance with the electron theory. Under continued application of electric force, there is a steady drift of electrons away from the negative towards the positive plates, thus giving surface values of resistivity dependent on the value and direction of the field, in accordance with the suggestion above.

The charges Q_1 and Q_2 on the first and second plates are

$$Q_1 = \frac{C}{4\pi}K_1\lambda_1, \quad \text{and} \quad Q_2 = -\frac{C}{4\pi}K_0\lambda_0 \tag{24}$$

and so

$$\frac{q}{Q_1} = \frac{K_0\lambda_0 - K_1\lambda_1}{K_1\lambda_1} \tag{25}$$

and if, as appears probable in a homogeneous dielectric, $K_1 = K_0$ and the specific resistivity is inversely proportional to the electron density, we have

$$\frac{q}{Q_1} = \frac{\lambda_0 - \lambda_1}{\lambda_1} = \frac{n_1 - n_0}{n_0}; \tag{26}$$

According to this view the residual charge is due to changes of the surface resistances consequent on a displacement of the free electrons under the action of the field of force. When the field is removed by connecting the plates, the residual charge disappears gradually and n_1 becomes equal to n_0.

In a series of experiments on sulphur, the ratio of residual to initial charge was found to depend on the potential difference V and to obey closely the law

$$\frac{q}{Q} = \frac{0.05V}{V + 10} \tag{27}$$

and thus to approach the value 0.05 at very high values of V. This leads to the conclusion that even for very high values of V the ratio $n_0:n_1$ is not less than 0.95, that is, only a relatively small disturbance of the normal electron density is necessary to account for the residual charge as observed.

THEORIES BASED ON ANOMALIES OF CONDUCTIVITY

Group 4

We now come to the theories of the fourth class, namely, those based on anomalies of conductivity. In reality there are no theories in this class sufficiently developed to merit the name. There are, however, many peculiarities of dielectric conductivity, which indicate departures from fundamental laws, and in which the time enters as a factor. It is natural, therefore, that these peculiarities should receive consideration as possible anomalies. Many investigators have studied our problem from this point of view. While it cannot be stated that any single well-formed theory has resulted from these studies, many of the facts observed are so important that our discussion would not be complete without giving them a prominent place.

It has often been suggested that dielectric absorption is due to a varying conductivity resulting from a motion of movable charges or "ions" within the dielectric; we have just noted an instance of this in the work of Anderson and Keane. In the case of solids there are obvious difficulties in this point of view. Instinctively we think of the conductivity of solid dielectrics as being of the same general nature as that in metals, into which an influence of time does not enter. Experimental investigations as to the exact nature of the conductivity of solid dielectrics are very difficult. As a consequence, principal attention has been directed to those other forms of conductivity which are found in liquids and gases, and in which the relations between current and electromotive vary with time. Many composite solid dielectrics reveal an "irreversible" anomalous current, that is, a slowly decaying charging current which does not reappear on discharge. This property is also possessed by most liquid dielectrics. Moreover, it is their only prominent anomalous property, residual charge being entirely absent. They thus afford excellent opportunity for the study of the irreversible anomalous current. We have already noted that this current does not obey Ohm's law, and that the apparent conductivity falls off with increasing voltage. This is also the case in an ionized gas, and this has led to the suggestion that the conductivity of both liquid and solid dielectrics is of the same nature as that in gases.

In the case of liquids the results of experiment are in fairly good qualitative agreement with the theory of gaseous conductivity. With the continued application of electric force, the ions are swept to the electrodes and the current falls off. With greater values of electric force, the ions are drained off more and more rapidly as compared with their rate of formation, and so the final conductivity falls off. The theory applies especially well to liquids having very low conductivity, in which an increased conductivity is induced by radioactive influence. In this case the liquid shows the characteristic behavior of ionized gases, *e.g.*, saturation current, etc., and methods have been worked out for the determination of the mobilities of the positive and negative ions, coefficients of recombination and other important quantities. Thus in many cases of liquids, carefully prepared and free from impurities, the laws of gaseous conduction serve quite well to explain that portion of the charging current which changes with time. The case is not so clear, however, for the final or inherent conductivity of the liquid, for this shows marked departures from the laws of gases. These departures are at times found to be due to an influence of the electrodes, or to dissolved or free impurities. But even with these removed, the laws of gases in their simple form are not obeyed, and in order to bring the results of experiment into conformity with the theory, it is necessary to assume a number of different types of ion, having in general different values of mobility. There appears no reason why these should not be present, but experimental evidence is wanting. Thus the laws of gaseous conduction explain that in liquids only in an approximate manner.

Efforts have been made also to extend the same explanation (*i.e.*, ionic conduction) to the anomalous charge and discharge currents of solids, but with little success. Von Schweidler has maintained that in an ionized medium the reversible anomalous current cannot arise, and also that the principle of superposition cannot be explained in this way; and he offers as proof the absence of residual charge in liquid dielectrics. On the other hand, as we have already noted, Anderson and Keane have shown that the drifting of free electrons to the positive electrode will result in a space variation of charge sufficient to account for residual charge in accordance with Maxwell's theory, and have checked their conclusions with observations on sulphur.

The behavior of glass is especially significant in this connection. The final conductivity of glass has undoubtedly an electrolytic character. Sodium is deposited out of glass on the electrodes, with resulting decrease of conductivity, and the latter may be maintained by providing an anode of sodium amalgam. Lithium may also be used, but apparently no other metals of the same chemical group. Moreover, the electrolytic action is in accordance with Faraday's laws. It must be remembered, however, that glass is not a homogeneous substance and, in fact, is generally considered to be a solid solution and so of highly special character. On the other hand, similar behavior has been observed in quartz, certainly not a solution, but in which small quantities of sodium and lithium are usually present. Mercury, quartz, and sodium form an electrolytic cell giving about 0.5 volt. H. H. Poole has shown the relation $\lambda = e^{a+bX}$ for the conductivity of glass, a being the thickness and X the field intensity; *i.e.*, the conductivity increases with the field strength. Günther-Schulze sees in this an evidence of ionic conductivity and ionization by collision as in gases. Thus, while we may not be sure that glass presents a behavior typical of all anomalous dielectrics, it unquestionably is significant as showing the possibility of ionic conductivity in solids.

Perhaps one of the most significant facts under this heading is the close relationship which exists between the final conductivity and the anomalous charging current curve, as affected by electromotive force and temperature. In all dielectrics well freed of moisture, these two properties appear to vary absolutely in unison. The final conductivity is obtained by subtracting from the charging current curve corresponding ordinates of the discharge curve; the result is generally a horizontal line representing the constant conductivity of the material. Under variation of electromotive force proportionate changes in both absorption and conductivity result. Under temperature variation, the final conductivity approximately obeys the relation $\lambda_\theta = \lambda_0 e^{a\theta}$ with corresponding change in the ordinates of the absorption curve. In these cases the evidence is very direct that absorption and conductivity are definitely related properties.

The Influence of Moisture.—Moisture even in the smallest quantity has a profound influence on the fundamental properties of dielectrics, and on their behavior when used as insulators. All porous and fibrous dielectrics absorb moisture rapidly from the surrounding atmosphere. Once taken up, the moisture leaves the dielectric but slowly and only under elevation of temperature. It is retained by the forces of surface tension in the capillary fibers and spaces of the material. H. L. Curtis exposed bakelite, fiber, marble, celluloid, and shellac to moist air 90 per cent saturated and then reduced the saturation of the surrounding air to 25 per cent. After standing for 40 days in the drier air, the dielectric properties of all the substances except shellac were still changing. Cable paper in the open absorbs up to 10 or even 20 per cent by weight of moisture. The insulation of all electric machinery is heated, impregnated, and varnished in order to drive off and prevent the subsequent absorption of water. Complete exclusion of water probably is never attained. Careful experiments on cable paper have shown that, as its temperature is raised to higher and higher values, water vapor is still given off up to the point at which the paper begins to disintegrate under the high temperature

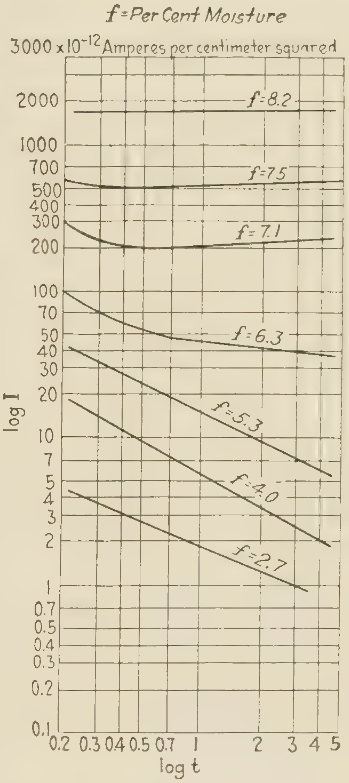
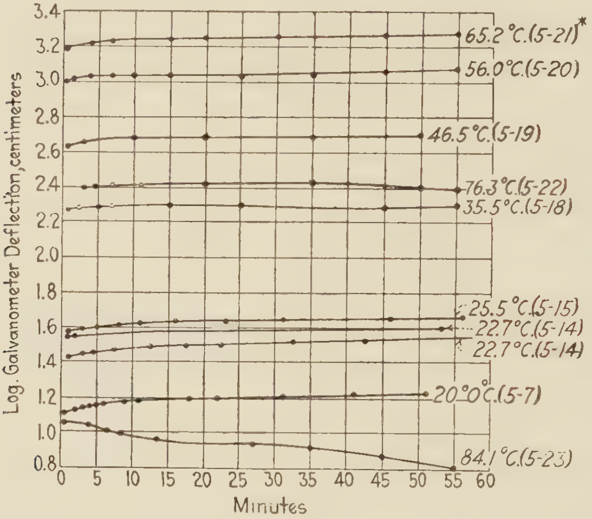


FIG. 20.—Current-time curves of cable paper as affected by moisture. (*Lubben.*)

The influence of moisture on dielectrics is best seen in their electrical behavior under continuous potential difference. The current under constant potential difference changes with the time, and the final value of the resistance varies with the potential difference. Cable paper, for example, when comparatively dry (2 to 3 per cent moisture content) shows a typical absorption curve in accordance with the formula $i_1{}^1 = at^{-m}$. With increasing moisture the ordinates increase, and the curve flattens out and becomes horizontal, except for a short initial descending portion, at about 7 per cent moisture content. Above this the curve rises gradually, *i.e.*, the charging current increases with time. Figure 20 shows curves from the observations of C. Lübben on flat sheets of cable paper between electrodes, showing these effects.

Figure 21 shows similar curves observed by the author on a sample of paper insulated high voltage cable before impregnation. In the latter studies the samples were prepared under ordinary atmospheric conditions and measured in the steady state at each of the temperatures as indicated. It will be seen that between the original state corresponding to about 10 per cent moisture content, and the final state, the final resistance changes 10 times. These increases of current with increasing moisture are almost entirely due to increased conductivity. The discharge current does not take on corresponding increases, in fact, remains of about the same order of magnitude, and follows approximately the equation given above, in all cases (see Fig. 20, Lübben and Fig. 21, the author). Thus the difference between charge and discharge currents at any instant which determines the insulation resistance increases with increasing moisture content. Moreover, above a certain value of moisture content, the difference increases with the elapsed time after the application of voltage. These facts indicate that the continued application of voltage also causes a decrease in resistance.



*(5-21) indicates date e.i. May 21st.

FIG. 21.—The influence of temperature on the conductivity of cable paper.

It is found, moreover, that the final steady value of resistance decreases with increasing value of the applied voltage. Figure 22 shows the variation of final resistance with voltage for cotton insulation as reported by K. W. Wagner. Figure 23 shows the values of current for increasing values of voltage, at different time intervals after the application of voltage. It is seen that the current in all cases increases more rapidly than the voltage. The observations were made on the paper and mica insulation of a 6,600-volt armature.

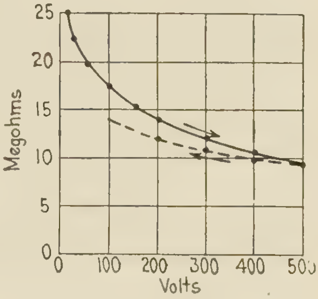


FIG. 22.—The influence of the value of voltage on the final resistance of cotton insulation.

The following approximate relations for the resistance r, the conductance g, and the conduction current i have been found to obtain for a fibrous dielectric containing moisture.

$$r = \frac{A}{\sqrt{E}}; \quad g = g_0 E^m; \quad i = i_0 t^p \tag{28}$$

E being the voltage and A, g_0, m, i_0 and p constants. Thus Eq. (2), Lecture II, for the total current on charge becomes

$$I_1(t) = i_1(t) + i_g t^p + i_a t^{-m} \tag{29}$$

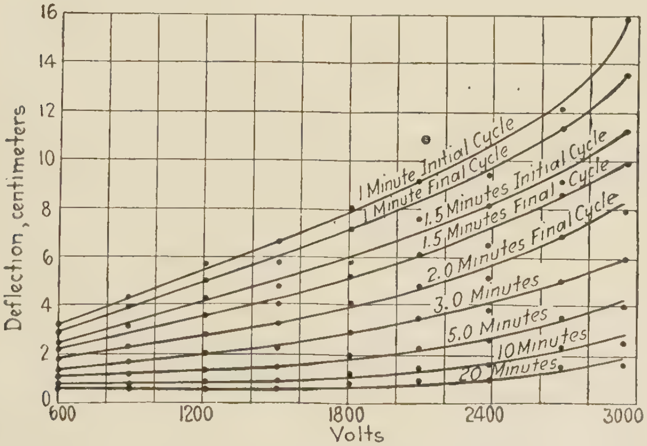


FIG. 23.—The influence of the value of voltage on the absorption of paper and mica insulation.

The second term is the conduction current due to moisture and is often negligible for low moisture content. The third term is the absorption current of the material in the approximately dry state.

In fibrous materials some moisture is always present, and the absorption term and the type of its variation are probably due in large measure to this residual moisture. The third term of the equation is completely obscured at high moisture content. The discharge current does not contain the second term.

An ingenious explanation of some of the above relations has been offered by E. Evershed. He makes careful determinations of the insulation resistance of clean filter paper which has absorbed different known amounts of water. The values found indicate that only a small portion of the water is active in lowering the resistance. Evershed thus supposes that the greater part of the water is held as drops in isolated openings in the insulating material, but that a part of it finds its way into a network of capillary fibers of the material. The water in these capillaries is separated by air bubbles, but the walls of the tubes surrounding the air bubbles are wet with thin films of water. These films constitute the principal resistance of the complete water path. Under the electric field, water is forced from the drops into the films, making their walls thicker and so increasing their conductivity. Figure 24 indicates the distribution of the water as suggested. In the thin state of the films they are very sensitive to the addition of more water, but less so as the walls get thicker, corresponding to the results of experiment. Evershed has supported his theory by a series of beautiful experiments on very small glass capillary tubes. Using a model consisting of a number of such tubes, he found that it gave the typical resistance, voltage, and time relations as found in fibrous insulation contain-

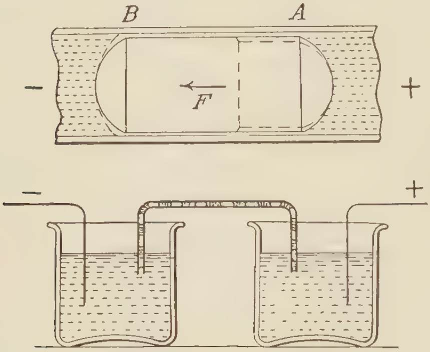


FIG. 24.—Evershed's model of a fibrous insulation containing moisture.

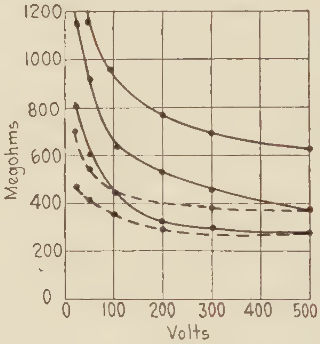


FIG. 25.—Voltage-resistance curves of the Evershed model.

ing small amounts of moisture. Figure 25 shows the resistance-voltage curves taken on the model, and also the influence of the time. The water moves into the tubes but slowly, and the influence of voltage, once exerted, persists for some time. The water films were observed under a microscope and found to behave as described above.

It should be noted that Evershed's theory deals only with the variable conductivity of dielectrics containing moisture. It offers no explanation of the fundamental property of absorption. This is no difficulty, however, for in accordance with the work of Wagner we have all the necessary elements for absorption when we picture a dielectric material with isolated conducting filaments of water embedded in it. In fact D. DuBois has proposed that all dielectric phenomena including those of alternating loss may be accounted for by discrete filaments of embedded moisture changing their shape and position under the alternating field.

That water may, in fact, play an important role in materials which contain no fibers but which are porous has been shown by J. Curie. By maintaining porcelain at difference degrees of moisture, he reproduced different types of charging-current curve, among them typical absorption curves as observed for dry substances. Moreover, this moist porcelain polarized up to several hundred volts, gave typical discharge curves, and in fact obeyed the principle of superposition. Curie suggests in explanation the linking up of a number of internal individual electrolytic cells due to water and local conducting impurities. This suggestion obviously differs fundamentally from the others mentioned above, and in fact recalls the idea of mobile ions proposed by Anderson and Keane.

In conclusion, then, we see that moisture has a most important influence on dielectric behavior, and that it is very difficult to eliminate completely. It is probable that many of the differences that have been found among the results of different observers in studying dielectric absorption, and many of the deviations from the indications of theory, are due to the uncontrolled presence of moisture. While we cannot suppose that moisture accounts for all dielectric absorption, it is probable that it plays a predominating part in many of the properties of the composite dielectrics used in the insulation of electric conductors and machinery. We shall mention its influence again when treating the losses in dielectrics under alternating stress.

With this lecture we close our discussion of the theories which have been proposed to account for dielectric absorption and residual charge. We are compelled to admit that no single one of them is entirely satisfactory. Many of them are obviously imaginary structures erected in the effort to picture some mechanism, usually artificial, conforming to known experimental facts. Others are merely ingenious sequences of mathematical manipulation. Maxwell's theory undoubtedly makes the strongest appeal. This, however, is due rather to the fact that it invokes no new phenomenon or property of dielectrics, than to any certain experimental confirmation. Dielectrics behave qualitatively but not quantitatively in accordance with Maxwell's theory. Yet it retains its hold on the minds of physicists. If not Maxwell's theory, what else? In my opinion Maxwell's treatment of the layer dielectric proves beyond question that a mixture of dielectrics will show the usual phenomena of absorption and residual charge. On the other hand, it appears equally obvious that in reasonably simple and pure dielectrics the absorption observed cannot be accounted for by a direct application of Maxwell's conception. We are forced to fall back on the various types of anomalous conductivity thought to be due to small quantities of moisture and impurities. To account for *absorption* we must assume for these impurities an exceptional ionic or electronic activity resulting in a change in the values of the dielectric constant and the conductivity, which in effect brings about the conditions pictured by Maxwell. Carefully controlled experimental studies of Maxwell's theory should not be difficult and are badly needed.

LECTURE VI

DIELECTRIC BEHAVIOR UNDER ALTERNATING STRESS

PART I

General.—There are two conspicuous anomalous properties of dielectrics. The first of these, dielectric absorption, we have already discussed in the foregoing lectures. We shall now consider the second, namely, the energy loss which occurs in solid dielectrics when subjected to alternating potential difference.

The losses in condensers under alternating potential difference were first noted by Siemens in 1864. Since then they have been studied by many observers. These losses, as we shall see, are far greater than the values computed from the conductivity of the dielectric. In some of the earlier condensers they were great enough to cause the solid dielectric to melt. For a long time and even up to the present, it has been assumed that the cause of these losses is to be found in some special and unknown property of the dielectric. This property is usually spoken of as dielectric hysteresis, and it is assumed to reside in some undiscovered molecular phenomenon similar to that in magnetic materials. We have already pointed out the objections to this point of view, and it is remarkable that it should have persisted for so long. As we are about to see, there is an obvious explanation of dielectric loss, if we accept the anomalous behavior of dielectrics discussed in the foregoing lectures. While, as we have just said, the explanation of dielectric loss is obvious, we must note at once that it is not yet possible to state definite laws connecting the losses in dielectrics with the simpler and more fundamental properties of the materials themselves. In fact, the problem of deriving the laws which govern these losses has probably received greater attention than any others in the field of dielectric behavior. The volume of the literature on dielectric losses is enormous. It embraces the studies of physicists whose chief interest is the ultimate explanation of the origin of the losses, and who for the most part have worked with pure materials, as well as the attacks of research engineers who have extended greatly the ranges of voltage and frequency, in the effort to improve the qualities of commercial insulation.

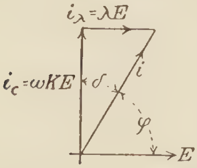


FIG. 26.—Vector diagram of a solid dielectric free from absorption.

Phase Difference in Dielectrics.—A condenser containing a perfect insulator as dielectric if subjected to alternating potential difference would consume no power. The charging current of the condenser would lead the difference of potential by exactly one-fourth of a period. The only condensers in which these conditions are approached are those with gases as dielectrics. All solid dielectrics have conductivity, and thus even if free from dielectric absorption they consume some energy. The vector diagram of such a condenser is shown in Fig. 26, in which E is the potential difference, I the total current, $I_g = \lambda_E$ the leakage current, $I_c = \omega KE$ the charging current, and cos Φ the power factor of the material due to conductivity alone. A condenser containing such a dielectric would have a value of capacity independent of the

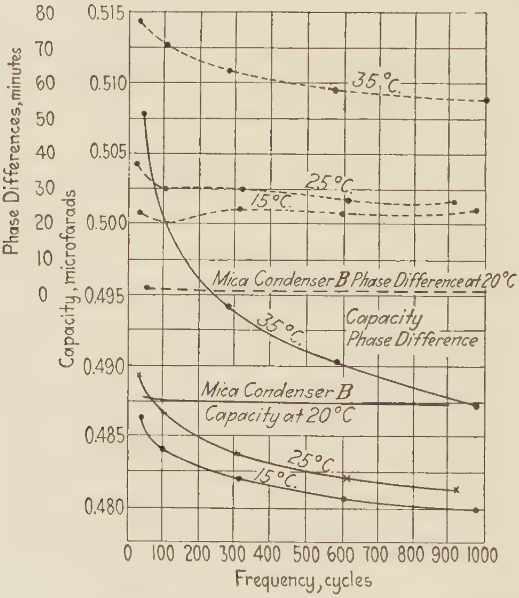


FIG. 27.—Capacity and phase difference as functions of the frequency in impregnated paper.

frequency and a power factor varying inversely as the frequency. Such dielectrics are unknown, however, and condensers made with the best insulating materials most carefully prepared all show power factors much higher than those corresponding to their conductivities and capacities much higher than their geometric values. Moreover, the power factors do not vary inversely as the frequency, but much less rapidly, and the capacity decreases slowly with the frequency. At times the power factor may increase with the frequency. Figures 27 and 28, as reported by F. W. Grover, show observations on commercial condensers in which the dielectrics were paper impregnated with paraffin. The comparative behavior of a standard mica condenser is also shown. It is evident from these curves that in the matters of capacity and phase difference the mica condenser is the only one in which the capacity seems to obey the fundamental law based on conductance alone, and all of them show departures in the matter of power factor.

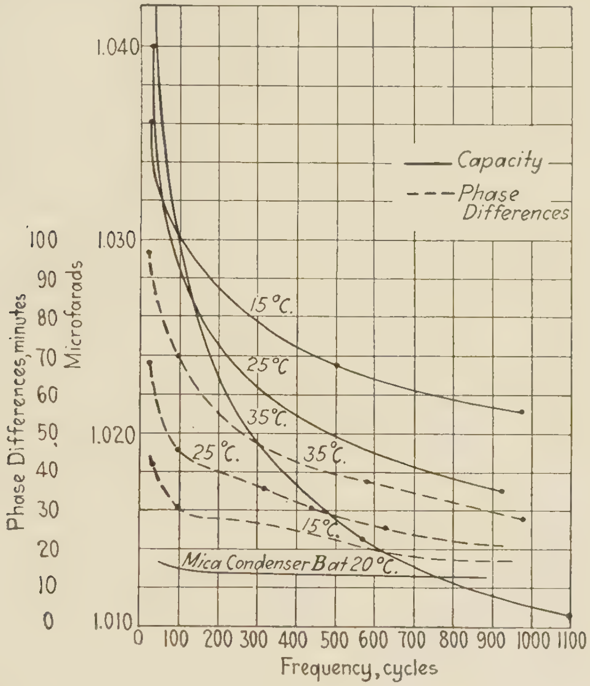


FIG. 28.—Capacity and phase difference as functions of the frequency in impregnated paper.

The term "power factor" in this case describes a property of the material and therefore is quite different in character and significance from the older connotation of the word. It is no doubt largely for this reason that the custom has arisen of describing this property of dielectrics not as the cosine of the angle of advance (angle Φ, Fig. 26) of the charging current over the applied potential or difference, but as the sine of the difference between the angle and 90 electrical degrees (angle δ, Fig. 26). This angular difference is called the "phase difference," or "phase defect." This custom has the added advantage that up to about 2 degrees the phase difference in radians, its sine, its tangent, and the power factor, in the usual sense, all have the same value within a very small fraction of 1 per cent. This makes it possible to use these several quantities indiscriminately, thereby greatly simplifying many computations. The use of the tangent of the phase difference is especially convenient, as it is the ratio of the energy component to the wattless component of the charging current (ratio $\frac{I_g}{I_c}$, Fig. 26).

Theories of the Origin of Dielectric Loss.—We have shown in Figs. 27 and 28 curves which are typical of many types of insulating material, although in different materials both the values and type of variation of the several quantities may vary widely. It is evident that there must also exist corresponding variations in the laws of the losses themselves. We shall first consider the more important theories which have been offered to explain dielectric loss, capacity, and phase difference, and thereafter we shall review briefly the extent to which these theories account for the behavior of insulating materials, as found in experiment and practice.

At the outset it may be said that the phenomenon of absorption is obviously sufficient to explain *qualitatively* the presence of a loss in dielectrics subjected to alternating potential difference. If absorption means the continued flow of current under a continued potential difference, then in the alternating case this must result in a lag of the charging current behind the applied difference of potential, and this is sufficient to account for an energy component of the current.

This fact has always been recognized by those who have investigated the phenomenon of dielectric loss. Thus Beaulard, in 1894, and Hess, in 1895, gave convincing arguments against the prevalent suggestion of dielectric hysteresis as underlying cause. Beaulard, in fact, concluded from his experimental studies that the losses arise in a dielectric viscosity. Hess concludes that Maxwell's theory of absorption is sufficient to account for dielectric losses. He conceived a simple model of a Maxwell dielectric consisting of two condensers in series, one having an infinite resistance and the other a definite value of conductivity. He investigated the behavior of this model under alternating potential difference and found that it accounted approximately for the results of experiment. Hopkinson, in 1897, measured the losses in condensers and attempted to find a connection between dielectric absorption and the values of capacity and dielectric loss. Ever since these early efforts many careful students of dielectric behavior have sought to explain these losses in terms of the phenomena of absorption as observed with continuous potentials. Just as there are a number of theories of the nature of dielectric absorption, so also there have been a number of different points of view as to the nature of dielectric losses, and analyses differing in accordance with these several theories have been offered from time to time. It will be recalled that in order to account for the results of experimental observation all theories of absorption find it necessary to assume the presence of a number of different materials, each of which introduces into the mathematical expressions its own terms involving its own set of electric constants. It is natural, therefore, that attempts to apply these fundamental theories directly to the alternating case have also encountered mathematical difficulties. There is, however, one circumstance which has simplified the problem in the alternating case. It is the fact that in the alternating case we are interested only in the steady electrical state of the material. In the case of continuous potential, absorption must be studied over very brief intervals of time, following the application or the removal of potential difference. In the alternating case it is only a question of maintaining external conditions constant, and one has plenty of time for measuring electric quantities.

We proceed to the review of the more striking of these analyses or applications of theories.

H. A. Rowland was apparently the first to make a rigid extension of Maxwell's theory of absorption to the alternating case. In 1897 he published the results of experiments on mica and impregnated paper condensers, traced their behavior to dielectric absorption, and derived values for the effective resistance and the effective capacity of Maxwell's layer dielectric. Both these quantities were found to involve the frequency and series of terms made up of the conductivities and the specific inductive capacities of the several layers. He did not present the analyses by which these values were reached, but this was subsequently done by F. W. Grover in 1911, checking closely the values of Rowland. The method followed by Grover was briefly as follows: Maxwell assumed that at any instant the total current in the external circuit and in each layer was the same, made up as follows:

$$i = \frac{X_1}{r_1} + \frac{K_1}{4\pi}\left(\frac{dX_1}{dt}\right) = \frac{X_2}{r_2} + \frac{K_2}{4\pi}\cdot\left(\frac{dX_2}{dt}\right) = \ . \ . \ . \qquad (1)$$

in which the X's are the potential gradients, the K's the dielectric constants, and the r's the specific resistances in the respective layers. If now in the steady alternating case the current is sinusoidal $i = I \sin pt$, we have

$$I \sin pt = \frac{X_1}{r_1} + \frac{K_1}{4\pi}\left(\frac{dX_1}{dt}\right) = \frac{X_2}{r_2} + \frac{K_2}{4\pi}\left(\frac{dX_2}{dt}\right) + \ . \ . \ . \qquad (2)$$

The electric force in any layer thus satisfies the equation

$$\frac{dX}{dt} + \frac{4\pi}{Kr}X = \frac{4\pi}{K} I \sin pt \qquad (3)$$

The solution of this equation contains the usual negative exponential term in the time t, and terms in sin pt and cos pt, from which may be derived an expression for the total electromotive force on the condenser involving three series of terms, one in $e^{-\alpha t}$, one in sin pt, one in cos pt.

The exponential term becomes negligible in a very short time, so that in the steady regime under alternating potential differences the electromotive force on the condenser terminals is of the form

$$e = IA \sin pt - IB \cos pt \qquad (4)$$

We may therefore write $e = E \sin (pt - \Psi)$ where $E^2 = M^2 + N^2$ and $\tan \Psi = \frac{M}{N}$, N and M being the coefficients of sin pt and cos pt respectively.

The *phase difference* is $\Phi = 90$ degrees $- \Psi$ and is found to be

$$\tan \Phi = pc\rho = \frac{N}{M}$$

$$\tan \Phi = pC\rho = \frac{N}{M} = \frac{4\pi}{p}\left[\frac{\dfrac{a_1}{K_1^2 r_1\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \dfrac{a_2}{K_2^2 r_2\left\{1+\left(\dfrac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \dots}{\dfrac{a_1}{K_1\left\{1+\left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \dfrac{a_2}{K_2\left\{1+\left(\dfrac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \dots}\right] \tag{5}$$

in which ρ is the equivalent resistance in series with the total capacity C.

Remembering that the component of the electromotive force lagging the current by one-quarter of a period is $\frac{I}{pC}$, we have

$$\frac{I}{pC} = IB \quad \therefore \quad \frac{1}{C} = pB$$

B to be taken from Eq. 4.

$$\therefore \quad \frac{1}{C} = 4\pi\left[\frac{a_1}{K_1\left\{1 + \left(\dfrac{4\pi}{K_1 r_1 p}\right)^2\right\}} + \frac{a_2}{K_2\left\{1 + \left(\dfrac{4\pi}{K_2 r_2 p}\right)^2\right\}} + \dots\right] \tag{6}$$

The value of ρ may be derived from Eqs. (5) and (6) and the resulting values are

$$\frac{1}{C} = 4\pi\left[\left(\frac{a_1}{K_1} + \frac{a_2}{K_2} + \dots\right) - \left(\frac{4\pi}{p}\right)^2\left(\frac{a_1}{K_1^3 r_1^2} + \frac{a_2}{K_2^3 r_2^2} + \dots\right) + \left(\frac{4\pi}{p}\right)^4\left(\frac{a_1}{K_1^5 r_1^4} + \frac{a_2}{K_2^5 r_2^4} + \dots\right) - \dots\right] \tag{7}$$

$$\rho = \left(\frac{4\pi}{p}\right)^2\left(\frac{a_1}{K_1^2 r_1} + \frac{a_2}{K_2^2 r_2} + \dots\right) - \left(\frac{4\pi}{p}\right)^4\left(\frac{a_1}{K_1^4 r_1^3} + \frac{a_2}{K_2^4 r_2^3} + \dots\right) + \dots \tag{8}$$

which are in substantial agreement with Rowland's expressions.

Rowland developed a sensitive dynamometer and many new forms of bridge connection, and with his coworkers made numerous studies of the behavior of dielectrics under alternating potential difference. He notes, however, that he could find in these measurements only a qualitative agreement with the expressions derived from Maxwell. Other workers have attempted to find agreement between the losses in dielectrics as observed and as called for by Maxwell's theory. Among them may be mentioned H. L. Curtis and F. W. Grover. Grover made extended observations on a number of condensers with values of frequency with the range 25 to 1,000 cycles, and at temperatures between 5 and 35 degrees Centigrade. He concluded that, on account of the number of variables involved and on account of uncertainty as to how many and what kinds of different materials were present, it was not possible to apply the results as a quantitative test of the theory. K. W. Wagner has studied a special case of Maxwell's theory, has compared it with the results of observation, and finds very good agreement in several particulars. We reserve his description of Wagner's analysis for a later paragraph.

Pellat and von Schweidler.—It will be recalled that Pellat assumed that when an electric field of strength E_0 is impressed on a dielectric, the displacement instantaneously takes up a value called for by fundamental electrostatic theory and then increases logarithmically toward a value $(1 + \epsilon)$ times as great, ϵ being small compared with unity. This assumption leads to the statement that the rate of increase of the displacement at any moment is proportional to the difference between the final value and that existing at the time in question or

$$D_t = \frac{KE_0}{4\pi}[1 + (1 - e^{-\alpha t})\epsilon]$$

$$D_\infty = \frac{K}{4\pi}E_0(1 + \epsilon)$$

$$\frac{dD_t}{dt} = \alpha e^{-\alpha t}\frac{\epsilon KE_0}{4\pi} = \alpha(D_\infty - D_t) \tag{9}$$

Extending this assumption to the case of a sinusoidal electromotive force, following von Schweidler, we note that for any law of the variation of the electric force $E_t = f(t)$

$$D_t = \frac{KE_t}{4\pi} + D_t' \tag{10}$$

where the instantaneous displacement D_t' is subject to Eq. (9), that is,

$$\frac{dD_t}{dt} = \alpha(D_\infty - D_t) \tag{11}$$

D_∞ being the final value of the displacement which would be reached if the electric field were held constant at the instantaneous value E_t. Remembering that

$$D_\infty = \frac{K}{4\pi} E_t(1 + \epsilon) \tag{12}$$

and using Eqs. (9) and (10), we have

$$\frac{dD_t'}{dt} = \alpha\left[\frac{K\epsilon}{4\pi}E_t - D_t'\right]$$

or

$$\frac{dD_t'}{dt} + \alpha D_t' = \frac{\alpha K\epsilon}{4\pi}E_t \tag{13}$$

Assuming now $E_t = E_0 \sin pt$,

$$\frac{dD_t'}{dt} + \alpha D_t' = \frac{K\alpha\epsilon}{4\pi}E_0 \sin pt \tag{14}$$

whose solution is

$$D_t' = Ae^{-\alpha t} + M \sin pt + N\, pt \tag{15}$$

The current is given by

$$\begin{aligned} i = \frac{dD_t}{dt} &= \frac{K}{4\pi}\left(\frac{dE_t}{dt}\right) + \frac{dD_t'}{dt} \\ &= \frac{pKE_0}{4\pi} \cos pt + \frac{dD_t'}{dt} \end{aligned} \tag{16}$$

The exponential term disappears in a short time and the current reduces to the form $i = A \cos pt + B \sin pt$, or $i = I \cos (pt - \Phi)$ in which Φ is the *phase difference.* The values of A and B lead to values of the phase difference and of the increase of the geometric capacity due to absorption. von Schweidler found that the expressions so found were not satisfied by the results of experiment, and Grover found that the same was true of his own experiments.

Since the elementary expressions indicated above do not account quantitatively for his measurements, von Schweidler proposes an extension of Pellat's theory and makes the assumption that the law of change of the absorbed charge, instead of being expressed by a single exponential term, may be represented by a series of such terms, namely,

$$D_t = \frac{KE_0}{4\pi}\left[1 + \sum \epsilon(1 - e^{-at})\right] \tag{17}$$

Carrying this new value of D_t through the same process as before von Schweidler reaches the values

$$\frac{\Delta C}{C} = \frac{\alpha_1^2\epsilon_1}{p^2 + \alpha_1^2} + \frac{\alpha_2^2\epsilon_2}{p^2 + \alpha_2^2} + \frac{\alpha_3^2\epsilon_3}{p^2 + \alpha_3^2} + \cdots \qquad (18)$$

$$\tan\Phi = \frac{\alpha_1\epsilon_1 p}{p^2 + \alpha_1^2(1 + \epsilon_1)} \frac{\alpha_2\epsilon_2 p}{p^2 + \alpha_2^2(1 + \epsilon_2)} + \cdots \qquad (19)$$

in which ΔC is the increase in the geometric capacity due to absorption.

It will be recalled that von Schweidler has proposed a type of physical structure of the dielectric which should account for a variation of the displacement in accordance with his assumptions. He assumes the presence in the dielectric of molecules with aperiodic damping under electric force. In order to conform to experimental observations on absorption, he found it necessary to assume several types of such molecules all obeying the same general law, but with different constants. Grover studied his own experimental results in the light of von Schweidler's expressions and concludes that three types of distribution, that is, three sets of constants in the formulae, give a reasonably good agreement with his observations.

von Schweidler's analysis must be considered only as an ingenious mathematical expedient, and his picture of a number of distributions of aperiodically damped molecules is highly artificial. Pellat's theory results in the same type of function for the variation of the anomalous charging current with the time, as does that of Maxwell. Consequently it would undoubtedly be possible to secure agreement between Maxwell's expressions and the experimental results of Grover, if sufficient attempt were made to find proper values of the thickness and the electric constants pertaining to a suitable number of layers. Grover, in fact, states that this difficulty prevented his attempting to check Maxwell's theory. It seems safe to say that any theory of dielectric absorption involving a charging current continually decreasing with the time would lead to agreement with the results of experiment with both continuous and alternating potential difference, if a sufficient number of terms, *i.e.*, a sufficient number of different dielectric materials, were assumed to be present. It does not appear possible, therefore, to find in von Schweidler's extension of Pellat's theory an agreement with the results of experiment which is not possessed in equal degree by Maxwell's theory.

Décombe.—In a foregoing lecture we have described a theory of absorption which was proposed by M. L. Décombe, and which finds the origin of all dielectric phenomena in the displacement of electrons within the atom. This permits him to use the general equation of electron displacement of Lorentz as starting point.

$$E = a\frac{d^2m}{dt^2} + c\frac{dm}{dt} + bm \tag{20}$$

in which E is the electric intensity, which may have the form $E = f\ (\omega t)$, m is the atomic polarization or electron displacement, and a, b, and c are constants of inertia, elasticity, and friction respectively. If q is the total displacement including that of the ether, and if the velocity of displacement is much less than that of light, one has two fundamental equations for absorption:

$$q = 4\pi E + m \tag{21}$$

$$E = bm + c\frac{dm}{dt} \tag{22}$$

Décombe extends this theory to the alternating case and in fact offers the alternating loss measurements on high voltage cables reported by Hochstaedter, as a principal support of his theory. His clear and concise discussion of this feature is as follows:

Lorsqu'on soumet un condensateur à lame diélectrique à une différence de potentiel alternative $E = f(\omega t)$, l'énergie élémentaire absorbée a pour expression Edq. Or, d'après l'équation (21), dq est de la forme $dm + k_0dE$, et, par suite, l'énergie absorbée élémentaire de la forme $E(dm + k_0dE)$.

Si nous integrons cette expression de 0 à T (T désignant la durée d'une période), nous obtenons pour l'énergie W dissipée pendant une période:

$$W = \int_0^T Edm \tag{23}$$

Si nous remplaçons alors dans l'expression de W la quantité E par sa valeur telle qu'elle résulte de l'équation fondamentale (22), nous obtenons:

$$W = \int_0^T c\left(\frac{dm}{dt}\right)^2 dt \tag{24}$$

La formule (23), qui exprime l'énergie dissipée pendant une période, nous montre que l'énergie dissipée élémentaire $c\left(\frac{dm}{dt}\right)^2$

dt est proportionnelle au carré du courant de polarisation. Ainsi se trouve rattachée aux équations fondamentales (21) et (22) l'hypothèse que j'ai précédemment proposée touchant l'origine et la valeur de la chaleur de Siemens. Celle-ci est attribuable à a polarisation des éléments de volume du diélectrique, c'est-à-dire à la portion du deplacement qui a son siége dans le diélectrique, et elle se présente comme proportionnelle au carré du courant de polarisation.

Mais on peut aller plus loin.

Les expériences de Höchstädter nous ont permis, en effet de mettre la polarisation m sous la forme:

$$m = \Gamma f(\omega t - \varphi).$$

Or, si nous développons la fonction périodique $f(\omega t - \varphi)$ suivant la série de Fourier et si, formant ensuite $\frac{dm}{dt}$, nous calculons, après substitution, l'intégrale (23), nous trouvons une expression de la forme:

$$W = cA\Gamma^2\omega \tag{25}$$

A désignant une constant.

Si l'on veut bien se rappeler que le coefficient c est inversement proportionnel à ω (formule 10), on voit que l'énergie W dissipée pendant une période est indépendante de la fréquence, résultat conforme aux expériences de Steinmetz et de Höchstädter et dans lequel on peut voir une nouvelle confirmation de nos équations fondamentales.

Ceci posé, remplacons effectivement dans (24) c par sa valeur $\frac{\varphi}{\Gamma\omega}$ tirée des formules (10). On obtient:

$$W = A\Gamma\varphi \tag{26}$$

L'énergie dissipée pendant une période est donc proportionnelle au décalage φ, pourvu bien entendu que ce décalage reste très petit. Dans les experiences de Höchstädter, cette condition est encore réalisée sous des differences de potentiel de 20,000 volts.

We see then that Décombe's theory of absorption based on electron displacement leads to three principal conclusions for the alternating case: (1) the loss per period is independent of the frequency, (2) is proportional to the phase difference φ of the polarization, and (3) is proportional to the square of the current due to the change in polarization, and so to the square of the electric intensity. In addition to the results of Hochstaedter and Stein-

metz, quoted by Décombe, a number of others have been reported, showing a total loss increasing with the frequency. Some of these, however, indicate a rate of increase less than proportional to the frequency, and Granier reports a loss per cycle due to absorption alone decreasing with increasing frequency. We shall see later that owing to the possible presence of losses of other types there is much confusion among the many experimental studies of the influence of frequency. The conclusion that the loss is proportional to the phase difference (for small values) is in good agreement with the results of experiment. Décombe finds that the results of Hochstaedter indicate a loss increasing more rapidly than as the square of the electric intensity. This is contrary to the result indicated by this theory. He goes to some length in a discussion of the frictional constant *c* to account for such a result. We shall see later, however, that most high voltage cables contain a certain amount of entrapped air, which is ionized above a certain potential gradient. This type of loss might well have accounted for Hochstaedter's results. Moreover, many other investigators have observed a loss proportional to the square of the electric intensity. This indication of the theory of Décombe, then, is in general accord with the results of observations.

We may say then of the theory of Décombe that it is very striking in its simplicity, that it is strengthened by the firm basis on which the electron theory now rests, and that it accounts in a general way for the behavior of dielectrics under alternating stress. We must not forget, however, that the basic assumptions of the theory do not account for the phenomena of residual charge, particularly the reversal of polarity on discharge as often observed. This must be considered a serious objection to this theory. If this difficulty could be removed and if the theory could be put into a form permitting experimental determination of the constants for single materials, it would immediately take a foremost place among dielectric theories.

LECTURE VII

DIELECTRIC BEHAVIOR UNDER ALTERNATING STRESS *(Continued)*

PART II

Maxwell's Theory as Extended by K. W. Wagner.—The experimental investigations and theoretical studies of dielectric behavior of K. W. Wagner have presented many results of importance. A follower of Maxwell, he probably has contributed more than any other investigator to the support of Maxwell's theory of dielectric absorption. In the beginning he was chiefly interested in the influence of the anomalous properties of dielectrics on the transmission characteristics of telegraph cables. This led him to his studies of the fundamental nature of dielectric absorption as we have reviewed it in an earlier lecture. His discussion of the alternating case, while not completely general, is nevertheless noteworthy for its simplicity and for the explanation it offers of a number of phenomena heretofore not well understood.

Briefly, instead of attempting to extend Maxwell's general equations to the alternating case, as done by Rowland and Grover, Wagner considers a Maxwell two layer dielectric and its behavior under alternating potential difference and in the steady regime. This method avoids the appearance of series of terms in the constants of the materials and permits the use of the complex imaginary expressions for potential difference, current, and impedance in the well-known manner. The results are in surprising agreement with those of experiment, and indicate clearly that, in accordance with Maxwell's theory, only two substances are necessary to account for many of the phenomena as observed. This leads to the further thought that if two fundamentally different substances are present, relatively small departures from the indications of theory may well be traced to small amounts of impurities. We should note in passing, however, that this does not account for the fact that in the cases of some pure dielectrics very small traces of impurities may occasion large changes in absorption.

The treatment of the two layer dielectric by Wagner, for the cases of both continuous and alternating potentials, starts from Maxwell's fundamental assumption of the simultaneous and parallel operations of the properties of conductivity and electrostatic induction, and the continuity of total current value throughout the condenser and the outside circuit. In the case of continuous potential, this condition is expressed by Eq. (5) of Lecture III, *i.e.*,

$$i_0 = X_n\lambda_n + \frac{K_n}{4\pi}\frac{dX_n}{dt} = i_n$$

In the alternating case and for unit area of a condenser having two layers of equal thickness d, we have in the steady state

$$i = \left(\lambda_1 + \frac{j\omega K_1}{y}\right)E_1 = \left(\lambda_2 + \frac{j\omega K_2}{y}\right)E_2 \tag{1}$$

which we may also write as

$$i = Y_1E_1 = Y_2E_2$$

i, E_1 and E_2 being complex, E_1 and E_2 being the electric intensities, λ_1 and λ_2 the conductivities, K_1 and K_2 the dielectric constants, Y_1 and Y_2 the apparent admittances in the respective layers, and y a constant converting from electrostatic to electromagnetic units $\left(\frac{1}{y} = 0.885 \times 10^{-13}\right)$.

The total potential difference will be of the form $E = E_0e^{j\omega t}$ and so we have

$$E_1 = \frac{E}{d}\frac{Y_2}{Y_1 + Y_2}; \quad E_2 = \frac{E}{d}\frac{Y_1}{Y_1 + Y_2}; \quad i = \frac{E}{d}\frac{Y_1Y_2}{Y_1 + Y_2}; \tag{2}$$

$$Y = \frac{Y_1\,Y_2}{Y_1 + Y_2} \tag{3}$$

is the apparent specific admittance of an equivalent homogeneous material, *i.e.*, a material which in a condenser of the same dimensions would show the same properties as the two layer condenser. Also

$$Y = \frac{\left(\lambda_1 + j\frac{\omega K_1}{y}\right)\left(\lambda_2 + j\frac{\omega K_2}{y}\right)}{\lambda_1 + \lambda_2 + j\omega\frac{K_1 + K_2}{y}} \tag{3a}$$

For continuous potentials, $\omega = 0$, Y reduces to λ, the specific conductivity of the equivalent material

$$Y_0 = \lambda = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2} \tag{3b}$$

For very high frequencies, λ may be neglected in comparison with $\omega\frac{K}{y}$, and we have

$$Y_\infty = j\frac{\omega}{y}\frac{K_1K_2}{K_1 + K_2} = j\omega\frac{K}{y} \tag{3c}$$

in which

$$K = \frac{K_1K_2}{K_1 + K_2} \tag{3d}$$

the resultant dielectric constant at high frequencies, which is seen to be the same as that of the equivalent material under continuous potential. If we write Y in the form

$$Y = \lambda + \frac{j\omega K}{y} + Y_n \tag{3e}$$

Y_n is seen to be the component of the apparent specific admittance, which is caused by the presence of dielectric absorption, for if absorption is not present, the condenser must show the behavior of a homogeneous material of specific admittance represented by the first two terms of the above expression.

We have then

$$Y_n = \frac{\left(\lambda_1 + j\omega\frac{K_1}{y}\right)\left(\lambda_2 + j\omega\frac{K_2}{y}\right)}{\lambda_1 + \lambda_2 + j\omega\frac{K_1 + K_2}{y}} - \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2} - \frac{j\omega}{y}\frac{K_1K_2}{K_1 + K_2}$$

which reduces to

$$Y_n = \frac{j\omega(K_1\lambda_2 - K_2\lambda_1)}{y(1 + j\omega T)(\lambda_1 + \lambda^2)^2(K_1 + K_2)} \tag{3f}$$

in which

$$T = \frac{K_1 + K_2}{y(\lambda_1 + \lambda_2)} \tag{3g}$$

is the "time constant" of the two layer condenser, which determines the rate of decay of the reversible anomalous current, *i.e.*, the charging current of the condenser, under continuous potential.

This value of Y_n indicates that for the case $\lambda_1K_2 = \lambda_2K_1$, $Y_n = 0$ and our two layer condenser shows no anomalies and, in fact, behaves as a condenser with single homogeneous dielectric. This is to be expected, since in accordance with Maxwell's theory there is no dielectric absorption in a two layer dielectric for which $\lambda_1K_2 = \lambda_2K_1$.

We may also conveniently write

$$Y_n = \frac{j\omega K}{y} \cdot \frac{h}{1 + j\omega T} \tag{3h}$$

in which, by Eqs. (3*d*) and (3*f*),

$$h = \frac{K_1\lambda_2 - K_2\lambda_1}{K_1K_2(\lambda_1 + \lambda_2)^2} \tag{3i}$$

is seen to be also a constant of the materials and is called by Wagner, the "absorption constant." Separating the real and imaginary components of Y_n we have

$$Y_n = \frac{j\omega}{y} \cdot \frac{hK}{1 + \omega^2T^2} + \frac{\omega^2KhT}{y(1 + \omega^2T^2)} \tag{3j}$$

Finally, substituting in Eq. (3*e*), we have for the apparent specific admittance of the equivalent homogeneous dielectric material

$$Y = \lambda + \frac{\omega^2KhT}{y(1 + \omega^2T^2)} + \frac{j\omega K}{y}\left(1 + \frac{h}{1 + \omega^2T^2}\right) \tag{4}$$

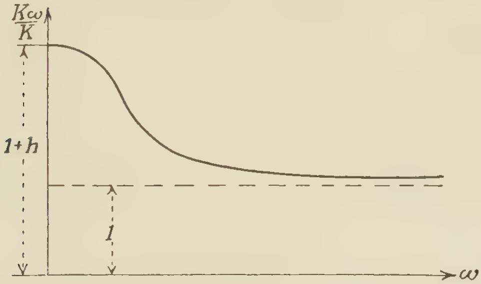


FIG. 29.—Two-layer dielectrics. Influence of frequency on apparent dielectric constant.

The real part of Eq. (4) determines the component of current in phase with the applied voltage and thus the energy loss. This loss is seen to be greater than that corresponding to the conductivity alone, as is well known.

The imaginary part of Y determines the charging current. It is obvious that in this case also the effect of absorption is to give an apparent increase to the dielectric constant,

$$K_w = K\left(1 + \frac{h}{1 + \omega^2T^2}\right) \tag{5}$$

K_ω decreases continuously with increasing frequency, from the continuous current value $K(1 + h)$ to the value K at very high frequencies in the manner shown in Fig. 29. Obviously the capacity of the condenser follows corresponding changes.

The current per unit area may now be written

$$i = \frac{E}{d}Y = \frac{E}{d}\lambda + \frac{E}{d}\,\frac{\omega^2 KhT}{y(1 + \omega^2 T^2)} + j\omega E\frac{K_w}{yd} \qquad (5a)$$

Thus $\frac{\lambda}{a}$ is the continuous current conductivity.

$$\frac{\omega^2 hKT}{dy(1 + \omega^2 T^2)}$$

is the increase in effective conductivity caused by absorption, at frequency ω, and $\frac{K_\omega}{yd}$ is the capacity for alternating current of frequency ω, all per unit area of electrode surface.

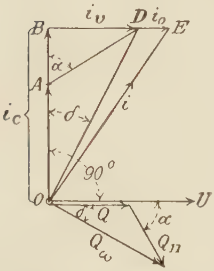


Fig. 30.—Vector diagram of a dielectric possessing absorption.

The total current i then is seen to consist of three components, i_0 the pure conduction current, i_v the dielectric loss current, and i_c the capacity charging current. They are combined as indicated in the vector diagram of Fig. 30. OB is the charging current i_c.

$$i_c = \frac{\omega E K\omega}{yd}$$

and is perpendicular to the voltage vector E; the part OA, or

$$i_c' = \frac{\omega E K}{yd}$$

would be the charging current if there were no absorption. The increase in the charging current by the amount AB is due to the penetration of charge into the interior of the dielectric. Since this charge must pass over the internal resistance of the dielectric, there necessarily results watt component $i_v = BD$. Finally there is the pure conduction current $\frac{E\lambda}{d}$ which is in phase with E and is plotted as DE: this component is usually extremely small as compared with BD and so may be neglected. The lower part of the diagram shows the relation of Q the vector representing the pure displacement charge, Q_n the residual charge, and Q the total charge. The angle δ by which the total charge lags behind the voltage is the angle of "phase difference," or angle of dielectric loss.

The power factor of the condenser neglecting i_0 is seen to be

$$\cos (90^\circ - \delta) = \sin \delta \approx \tan \delta = \frac{i_v}{i_c}; \tag{6}$$

or

$$\tan \delta = h \frac{K}{K_w} \cdot \frac{\omega T}{1 + \omega^2 T^2}; \tag{6a}$$

$$= h \frac{\omega T}{1 + h + \omega^2 T^2} \tag{6b}$$

Figure 31 shows the variation of the angle of phase difference with the frequency. It reaches a maximum value $\frac{h}{2}\sqrt{1 + h}$ when $\omega T = \sqrt{1 + h}$. Moreover, if G is the effective conductivity and C the effective capacity.

$$i_v = GE\,;\; i_c = WCE;\; \tan \delta = \frac{G}{WC} \tag{7}$$

and these equations determine the relation between the angle of phase difference, the conductivity, and the capacity for alternating currents.

The power loss then is

$$W = Ei_v = GE^2 = \omega CE^2 \tan \delta; \tag{8}$$

that is, the dielectric loss varies as the square of the applied voltage and as the tangent of the angle of phase difference.

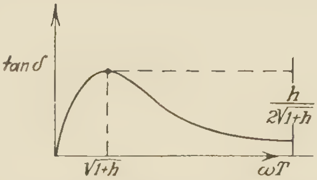


FIG. 31.—Two-layer dielectric. Influence of frequency on phase difference.

Comparing the foregoing relations with the results of experiment, we find that numerous studies of the relation between dielectric loss and voltage indicate that the former varies as the square of the latter. Many results have been reported in which this relationship is departed from to some extent, but the almost invariable conclusion is that the loss varies as some power of the voltage in the neighborhood of 2.

As regards the influence of frequency on the power factor and on dielectric loss, there has been much apparent contradiction among the results of different observers. Some have found that the power factor increased with the frequency, and others that it decreased. The curve of Fig. 31 shows how this is possible, the power factor or phase difference passing through a maximum at a frequency which varies with the constant h. As an example, Wagner shows that for a combination of paper and mica the frequency at which maximum loss increases will be in the neighborhood of 4 cycles per second, whereas if the mica were replaced by a substance having relatively high dielectric constant and high conductivity, the frequency at which maximum power-factor losses would occur might rise to 100,000 cycles or more. In the former case, the value of the equivalent conductivity due to absorption is found to be 5,330 times that of the continuous current conductivity λ, thus showing the relatively small part played by the leakage conductance. Considering these two examples, if measurements were made in the usual range of commercial power frequencies, the first would show the power factor decreasing, and the second the power factor increasing with the frequency. Figure 32 shows observations reported by Wagner in which tan δ, that is, the phase difference, or power factor, is shown passing through its maximum with increasing frequency. The decreasing capacity with increasing frequency is also evident.

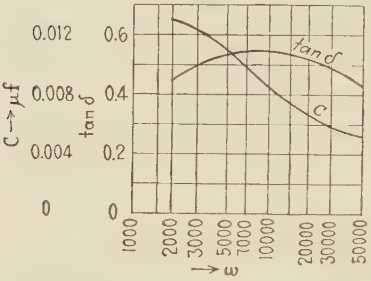


Fig. 32.—Impreganted cotton. Influence of frequency on capacity and phase difference.

Temperature variations are known to cause wide changes in the properties of dielectrics as observed under alternating voltages. They account for many of the apparent contradictions and inconsistencies in experimental studies. Some of these may be explained, at least qualitatively, in terms of the formulae derived above for the two-layer dielectric. The influence of temperature is mainly to be found in the variation of the conductivity λ. The conductivity of all dielectrics, particularly those of composite and fibrous character, increases greatly with the temperature. The dielectric constant, however, changes in only very small amount. Consequently the time constant T

$$T = \frac{K_1 + K_2}{y(\lambda_1 + \lambda_2)}$$

decreases rapidly with increasing temperature. On the other hand, the absorption constant h will not change greatly with the temperature provided that either one of the conductivities λ_1 or λ_2

is considerably greater than the other. This may readily be seen by inspection of Eq. (31). This is usually the case in the composite materials of commercial insulation. Now the time constant T determines the rapidity with which the charging current decays under continuous potential, and also the dependence of the power factor $\tan \delta$ on the frequency under alternating potential. The absorption constant h, however, determines the strength of the absorption current and the maximum value of the power factor.

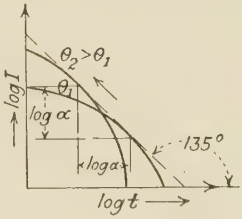


Fig. 33.—Two-layer dielectric. Charging-current temperature curves as based on theory.

From the standpoint of absorption under continuous potentials, we conclude that with an increase of temperature the total absorbed charge remains practically constant, but the rate of the decrease of charging current increases. Theoretical curves based on this reasoning are shown in Fig. 33 and curves observed by Wagner on an impregnated paper condenser in Fig. 34.

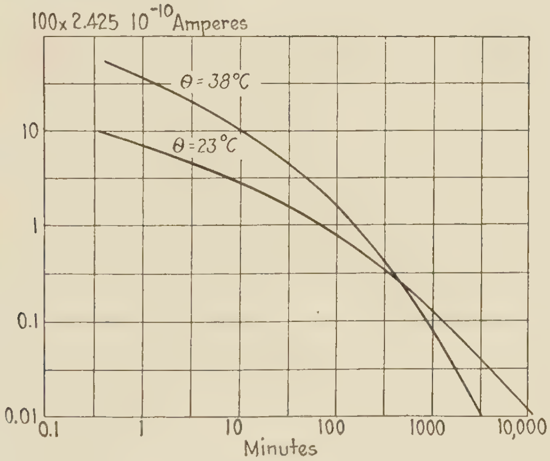


Fig. 34.—Curves of charging-current temperature as observed in impregnated paper.

Equally interesting qualitative agreements between theory and observation are shown for the relation between dielectric loss and frequency. Figure 35 shows the relation between power factor and frequency at two different temperatures as computed by Wagner for the Maxwell two layer dielectric. The absorption constant h is assumed to be practically independent of the temperature as we have seen, and the time constant T decreases with temperature. Since in accordance with Eq. (6b) time constant and frequency appear in the value of the power factor only as the in-product ωT, we find that a change of the time constant T causes only a shifting of the curves parallel to the axis of frequency, the general form of the curves remaining the same, and the maximum value of the power factor remaining unchanged, as indicated in Fig. 35. If a number of curves of this kind are taken for different frequencies, they may be transformed into a corresponding family of curves plotted between tan δ, the power factor, and Θ, the temperature, for various frequencies. This latter type of curve is very much easier to investigate experimentally inasmuch as the changes in question are very much more sensitive to changes in temperature than they are to changes in frequency. Figure 36 shows this transformation of the curves plotted in Fig. 35, Fig. 37 shows observations on beeswax reported

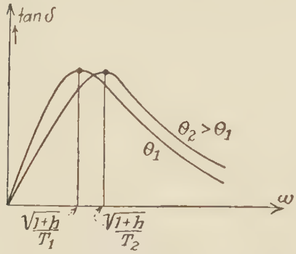


FIG. 35.—Two-layer dielectric. Relation between phase difference and frequency at two temperatures.

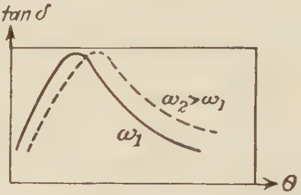


FIG. 36.—Two-layer dielectric. Relation between phase difference and temperature at two frequencies.

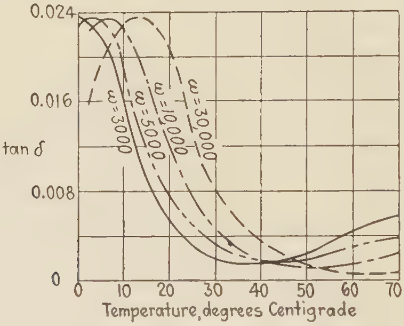


FIG. 37.—Relation between phase difference and temperature as observed in experiment.

by Wagner, and Fig. 38 others reported by Fleming and Dyke for gutta-percha. As will be seen, these curves show very clearly that at any one frequency changes in temperature cause very sharp changes in power factor, and that these changes may be either positive or negative, depending upon the absolute value of the temperature and the frequency at which the observations are made. The tendency indicated in Fig. 37 for the power-factor curves to rise again at higher temperatures is probably due to the fact that at a temperature in the neighborhood of 40° C. the substance undergoes a physical change. It will be seen from these results that the Maxwell two-layer dielectric affords qualitatively at least, a very satisfactory explanation of some of the more conspicuous features in the behavior of composite dielectrics under alternating potential difference.

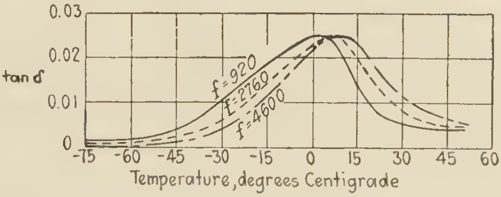


FIG. 38.—Relation between phase difference and temperature as observed in experiment.

On the other hand many other observations do not follow the comparatively simple relations indicated by Eqs. (5) and (6*b*). In order to account for these variations, Wagner introduces the idea, already described above, of a homogeneous dielectric with small conducting particles embedded in it. In an extended mathematical discussion, he arrives at the conclusion that such a combination behaves as does the Maxwell model with certain modifications of the constant. New expressions are derived for $K\omega$ and $\tan \delta$, in general, similar to Eqs. (5) and (6*b*) but also involving the relative quantity of the embedded conducting material, *i.e.*, impurity. Several such impurities may be present. Each will contribute its own effect, and their total may show curves differing markedly from those of Eqs. (5) and (6*b*). In order, therefore, to bring theory and observation into line, it is only necessary to assume a sufficient number of impurities present, and Wagner has been able to do this in several apparently complex cases, claiming that generally only a few sets of constants, *i.e.*, different impurities, will be necessary. Much the same types of discussion are possible with von Schweidler's picture, and also in adhering to Maxwell's original equations. In fact, Wagner's equations in several cases are readily transformed into those derived much earlier by Rowland, Curtis, and Grover.

Wagner's chief contributions to our knowledge of the behavior of dielectrics under alternating stress are his discussion of the variations of the phase difference, as related to the frequency and the temperature, and his experiments in support thereof. In this his work constitutes a strong support of Maxwell's theory. Nearly all so-called theories account qualitatively for experimental observations under continuous potential. Maxwell's theory as extended by Wagner is the only one in which for the alternating case the agreements mentioned have been shown.

LECTURE VIII

DIELECTRIC BEHAVIOR UNDER ALTERNATING STRESS (*Continued*)

Part III

The losses in dielectrics under alternating electric stress have a fundamental influence on the performance and life of many types of insulation. It is therefore of the first importance that the laws governing these losses be well understood. Unfortunately it cannot be said that a perfect understanding has been reached; nor even an understanding which permits the accurate design of insulation. It is reasonably certain, however, that the principal cause of the losses in insulation is the anomalous dielectric property which we have called "absorption." We have seen in the two foregoing lectures that several well-known theories of absorption, when extended to the alternating case, have yielded formulae more or less closely in agreement with the results of experiment. In these theoretical extensions the fundamental equations, developed to explain absorption as observed with continuous potential, have been retained. They are only modified by the substitution of the usual expression $E = E_0 \sin \omega t$ for the alternating potential difference. Thus, in the developments of Rowland and Wagner, these fundamental equations lead directly to expressions giving the energy loss, the phase difference, the equivalent capacity, etc., without special reference to the behavior of the dielectric under continuous potential. Such agreements with the results of observation as those reported by Wagner in his extension of Maxwell's theory, given in our last lecture, thus constitute partial support of the respective underlying theories.

Although many investigators have preferred to trace the origin of dielectric losses to some unknown form of internal viscosity or hysteresis, from the time of the first recognition by Siemens, in 1864, of the anomalous character of the losses, it has been suggested that the known presence of the property of absorption is sufficient to account for them. Hopkinson, Hess, Beaulard, conspicuous among the early investigators, all clearly emphasized the sufficiency of absorption to account for the losses, as we have seen. Absorption results in a sustained passage of current under a sustained continuous voltage and, therefore, must cause a component of current in phase with an alternating voltage.

Many other investigators of the laws of alternating dielectric behavior have been satisfied with this view, and have been content to link up the alternating with the continuous phenomena, leaving the theory as to the nature of the latter to the physicists. The most important studies in this class are those of von Schweidler, Jordan, and F. Tank on the one hand, and of the distinguished French physicist H. Bouasse on the other. The development of Bouasse has recently been extended by J. Lahouse. In each case these investigators have found in the principle of superposition a means of connecting the steady state of a dielectric under alternating voltage, with the transient or varying state under continuous voltage.

It will be recalled that the principle of superposition provides a means whereby the influence of the entire foregoing history of the dielectric, in causing residual charge, may be expressed for any present or future instant of time. The steady alternating case may therefore be studied by applying the principle of superposition, noting that an electromotive force of the form $E = E_0 \sin \omega t$ has been continuously applied for a long time in the past.

Von Schweidler, Jordan, Tank.—The most convenient expression for making use of the principle of superposition is that of the anomalous charging current (see Lecture II, Eq. (10)):

$$i(t) = BC\int_{-\infty} \frac{dE(u)}{du}\varphi(t - u) \cdot du$$

which may be written

$$i(t) = -BC\int^{\infty} \frac{dE(t - u)}{du}\varphi(u) \cdot du \qquad (1)$$

in which B is a constant, C the geometric capacity, t the instant at which i is measured, u the elapsed time controlling the variation of the voltage in the past, and $\varphi(t)$ is the function controlling the decay of the anomalous charging current under continuous potential suddenly applied to the dielectric after a long period of short circuit.

Applying to the alternating case, if $E = E_0 \sin \omega t$, in the steady state the current will have the form:

$$i(t) = \omega E_0 C(A \cos \omega t + D \sin \omega t) \tag{2}$$

Equating the two values of $i(t)$, we find

$$A = \int_0^\infty \varphi(u) \cos \omega u \cdot du$$
$$D = \int_0^\infty \varphi(u) \sin \omega u \cdot du \tag{3}$$

Thus remembering that the condenser has a leakage resistance R and a geometric capacity C, the total current in the steady alternating state is given by

$$I(t) = \omega C E_0 \left\{ (1 + A) \cos \omega t + \left(\frac{1}{\omega C R} + D \right) \sin \omega t \right\} \tag{4}$$

The apparent capacity is

$$C' = C(1 + A) \tag{5}$$

and neglecting the leakage conductance the loss is given by

$$W = \frac{1}{2} \omega C E_0^2 D \tag{6}$$

Now both C' and W can be measured, and so we have an experimental method for determining A and D, and a test of the theory that alternating dielectric loss is caused by absorption.

This is the method of von Schweidler. In order to make use of it, not only must $\varphi(t)$ be determined, but it must be of such a form as to permit integration of the Eqs. (3). Now, as is well known, $\varphi(t)$ is not always of simple form. In fact for most of the composite types of insulation it is not possible to write an expression for $\varphi(t)$. Hence this method at first appears to permit only very narrow application. H. Jordan and F. Tank have applied the above equations to observations both of absorption and of alternating losses in the same materials. The agreement found by Tank is noteworthy.

Recognizing the difficulty as to $\varphi(t)$, Tank has studied the general expression for $\varphi(t)$ as developed by von Schweidler for a complex dielectric. He reaches the conclusion that the curve for the general case is closely identical with that given by $\varphi(t) = \beta t^{-n}$, when $0 < n < 1$, and for values of t small compared with the period of the alternating voltage. This assumption greatly simplifies the evaluation of A and D in Eq. (3); in fact, von

Schweidler derived A and D for this case, *i.e.*, $\varphi(t) = \beta t^{-n}$. This form of $\varphi(t)$ has often been observed for much longer periods, as in the work of Hopkinson and Curie.

The principal value of Tank's work, however, is found in his experimental results. By means of the Helholtz pendulum he determined the form of $\varphi(t)$ for time intervals between 0.32×10^{-3} and 0.22 second, and found that for three out of six condensers studied the current closely followed the above law, with n in the neighborhood of 0.7. Adopting the corresponding form of $\varphi(t)$, the integrals of Eq. (3) are readily evaluated and the loss component of the current computed. He also measured the alternating losses in the same condensers by a sensitive electrodynamometer method and found a remarkable agreement between the measured and the computed values of losses and charging currents. The percentage differences between computed and measured values of loss were 3.8, 4.1, −0.4, 1.6, 13.8, and 23 per cent.

Tank also measured the anomalous charging current of a number of insulating liquids. He finds that they obey approximately the law $i = \beta t^{-n}$ for intervals between 1 and 15 minutes. All however have a definite maximum conductivity for short periods of application of voltage. This is shown in Fig. 39. This maximum is far less than the values indicated by the above expression. The measured values of alternating loss agree closely with values computed from this initial value of conductivity.

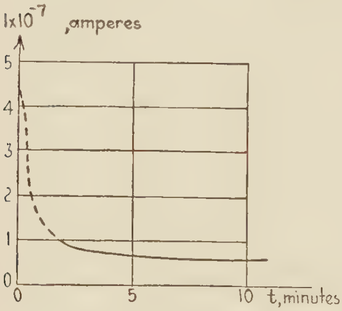


FIG. 39.—The charging current of a liquid dielectric.

The important conclusions from Tank's work are that the alternating losses in solid dielectrics are almost all accounted for by absorption; that the loss due to leakage conductance is only about 1 per cent of the total; and that there is no evidence of losses of other character. Owing to the simple form assumed and measured for the function $\varphi(t)$, this work may hardly be considered a confirmation of von Schweidler's theory as claimed by the author. However, it is apparently the only effort so far made for a direct check between measured loss and loss computed from measured absorption.

Bouasse.—In his notable work, "Cours de Magnetisme et d'Electricité," H. Bouasse, Professeur à la Faculté des Sciences de Toulouse, has clearly indicated our ignorance as to the fundamental character of the phenomenon of absorption. Therefore, in his too-brief treatment of the alternating case for dielectrics he limits himself to well-established experimental phenomena; of these he finds only two. The first is that any state of polarization set up in a dielectric persists for a certain time and disappears under some decreasing function of the time ($\varphi(t)$, Hopkinson's "factor of forgetfulness"). The second experimental fact is that the polarization resulting from any value field intensity is independent of foregoing polarizations. These two experimental facts together constitute the well-known principle of superposition.

It will be noted that in this restatement of the principle of superposition the intensity of polarization I is the *effect*, and the electric field E is the *cause*. This is in accordance with the view of Professor Bouasse that all of the anomalous properties of dielectrics should be traced to the polarization of the material of the medium as the seat of all the fundamental laws of dielectrics. For a description of his extension of the two simple experimental facts mentioned above to the alternating case, I feel that I cannot do better than to quote briefly his own words:

Au temps τ (compté vers le passe, à partir de l'époque actuelle prise pour origine) et pendant le temps $d\tau$, le champ était $P(\tau)$. Nous posons que le polarization actuelle I est due au champ actual augmenté de l'action (plus ou moins oubliée) des champs anterieurs.

$$\text{On a donc: } I = k\left[P + \int_0^\infty P(\tau) \cdot \phi(\tau) \cdot d\tau\right] \tag{7}$$

Exprimons $P(\tau)$ en fonction du temps t. Le phénomène qui se passe au temps τ dans le passé à partir du temps t actuel, se passe effectivement au temps absolu $t - \tau$.

En definitive nous écrirons au temps t:

$$I = k\left[P + \int_0^\infty P(t - \tau) . \phi(\tau) . d\tau\right] \tag{8}$$

La fonction ϕ doit être nulle pour $\tau = \infty$, avoir une valeur finie pour $\tau = 0$. Nous ferons nos calculs en posant, suivant les cas:

$$\phi(\tau) = \frac{a}{b + \tau} \cdot$$

$$\phi(\tau) = ae^{-b\tau}$$

For the alternating case it is only necessary to substitute $P = P_0 \sin \omega t$ in Eq. (8), which then becomes

$$I = kP_0\left[\sin \omega t + \int_0^\infty \sin \omega(t - \tau) \cdot \phi(\tau) \cdot d\tau\right] \tag{9}$$

Calculons l'intégrale de I:

$$I = \sin \omega t \int_0^\infty \cos \omega\tau \cdot \phi(\tau) d\tau - \cos \omega t \int_0^\infty \sin \omega\tau \cdot \phi(\tau) \cdot d\tau \tag{10}$$

Quelque soit le facteur d'oubli, les deux intégrales définies précédentes ont des valeurs finies bien déterminées. Nous avons donc:

$$I = kP_0[\sin \omega t + a \sin (\omega t - \beta)] \tag{11}$$

avec les conditions:

$$a \cos \beta = \int_0^\infty \cos \omega\tau \cdot \phi(\tau) \cdot d\tau, \qquad a \sin \beta = \int_0^\infty \sin \omega\tau \cdot \phi(\tau)\, d\tau$$

Le courbe qui relie I et P est encore une ellipse.

Pour faciliter les calculs et fixer les idées, prenons pour facteur d'oubli l'exponentielle:

$$\phi(\tau) = ae^{-b\tau} \tag{12}$$

On trouve (Math. Gen., 304)

$$\int_0^\infty \cos \omega\tau \cdot \exp(-b\tau) d\tau = \frac{b}{\omega^2 + b^2} = m \cos \beta$$

$$\int_0^\infty \cos \omega\tau \cdot \exp(-b\tau) d\tau = \frac{\omega}{\omega^2 + b^2} = m \sin \beta$$

D'ou

$$I = kP_0 [\sin \omega t + am \sin (\omega t - \beta)]$$

avec la condition:

$$tg\beta = \omega : b$$

As already stated, Eq. (11) is the equation of an ellipse between I and E_0. This result is independent of the form of $\varphi(t)$. Very beautiful examples of these ellipses have been found experimentally by Granier (see Fig. 40). The area of the ellipse may readily be found to have the value

$$w = \pi k a E_0^2 \sin \beta \tag{13}$$

that is, the loss per cycle is proportional to the square of the maximum potential difference as often observed.

It will be noted that, in order to obtain this relatively simple expression for the loss per cycle, it has been necessary to assume that Hopkinson's "factor of forgetfulness" is as given in Eq. (12).

This is also the form called for by Maxwell's development, and has been often observed in experiment.

J. Lahouse has used the foregoing method of Bouasse for deriving other approximate formulae, and an interesting discussion of the vector diagram covering the alternating case. He points out that if n in Eq. (12) is small as usually observed, the loss per cycle becomes

$$w = \frac{2k\omega m E_0^2}{\omega^2 + n^2} = \frac{2kmE_0^2}{\omega} \tag{14}$$

and the total loss is

$$W = \frac{kmE_0^2}{\pi} \tag{15}$$

that is, the loss per cycle is inversely proportional to the frequency, and the total loss is independent of the frequency. Equation (12) also indicates that for low frequencies, say a few cycles per second, w the loss per cycle has a maximum value for $n = \omega$. This behavior has been noted experimentally although the value of the frequency at which the maximum occurs sometimes has quite high values. We shall see below that there is some experimental support for a dielectric loss independent of the frequency, but the weight of the evidence shows a loss increasing with the frequency.

One should not attach too much importance to discrepancies between such a theory as that of Bouasse and the results of experiment. The theory is developed on the assumption of a single characteristic property of the dielectric material, namely, molecular polarization. Most experimental evidence is obtained from materials which may also contain other causes of dielectric loss. Many experiments have been made on commercial condensers having complex dielectrics, and most of these contain moisture and other impurities. All composite dielectrics, especially if they contain moisture, are subject to other types of loss than that due to polarization or viscosity. It is easy to imagine that some of these other causes may lead to losses which either increase or decrease with the frequency, and so to a variation other than that due to frequency alone.

J. Granier has investigated in every elegant manner the losses due to absorption alone, for alternating frequencies between 0.3 and 150 cycles. He interrupts the steady alternating excitation at different points on the cycle, and by a careful zero method obtains the total residual charge corresponding to each point.

This permits the plotting of the absorption voltage relation for a complete cycle at each frequency. Plotted in rectangular coordinates the curves are found to be almost perfect ellipses, as called for by the theoretical analysis of Bouasse (see Fig. 40). The area of the ellipse, representing the loss per cycle, continually decreases with increasing frequency. This is in general accord with the theory of Bouasse as extended by Lahouse. The ellipses become quite flat within the range mentioned, and the study of the influence of frequency is continued up to 1,500 cycles, using bridge methods. At still lower frequencies, according to the theory, we should also expect the ellipse to collapse into a straight line. Within the entire range investigated, the total loss increases, but less than in proportion to the frequency; the relation found is very consistent over the entire range and for several condensers studied is

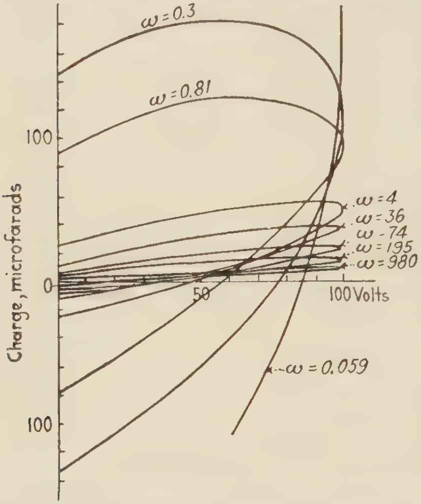


Fig. 40.—Variation of absorbed charge with voltage in an alternating cycle.

$$W = mf^n \qquad (16)$$

W being the loss *per second*, *f* the frequency and *m* and *n* constants. The following values of *n* were found: paraffin paper, 0.4; wax paper, 0.5; glass, 0.78; gutta-percha, 1 (in this case the phase difference being very small even at low frequencies). The results also show both the capacity and phase difference decreasing steadily over the whole range of frequency.

COMPARISON OF ALTERNATING THEORY AND EXPERIMENT

The above review indicates that, almost without exception, students of the theory of alternating dielectric losses agree in attributing these losses to the phenomenon of dielectric absorption. Moreover, although there are various theories as to the ultimate nature of absorption, they all conform fairly well to the results of experiment. This renders it difficult to make choice among them. But it is really unnecessary to do so, if we are concerned only with an explanation of the origin of alternating dielectric losses, for the results of the analyses of all the authors cited above agree quite closely as to the general laws governing loss, phase difference, and capacity, as influenced by voltage, frequency, and temperature, although they start in one or two cases from quite different premises.

So far as agreement between theory and the results of experiment goes, it will have been gathered that such agreement is approximate only and is to be found only in the general mode of variation of the several dielectric properties with the variation of external electric and other conditions. Absorption itself cannot be controlled or predicted, and so it is not to be expected that definite expressions and constants may be laid down for the behavior under alternating electric intensity. With this limitation in mind the following agreements between theory and experiment appear to place beyond question the seat of the principal part of the dielectric loss in solid dielectrics in the fundamental phenomenon of dielectric absorption.

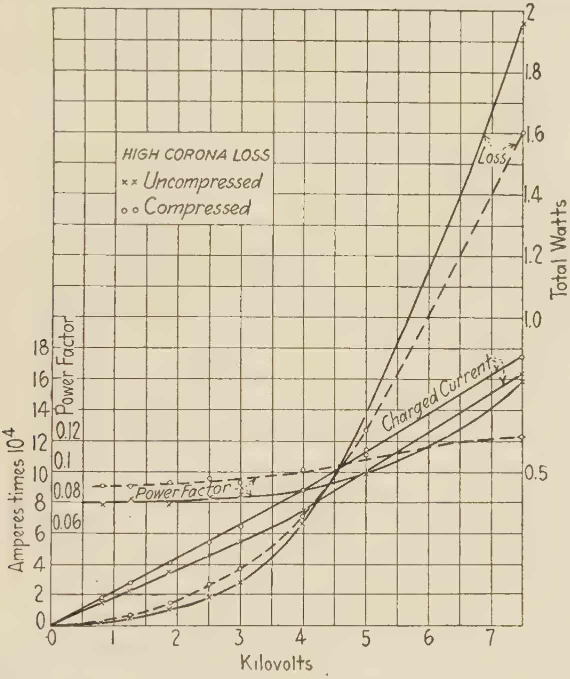


FIG. 41.—Variation of dielectric loss with voltage. Paper and mica.

Loss-voltage Curve.—There is almost universal theoretical agreement that the rate of loss varies as the square of the electric intensity. Many observers have studied this relation (see bibliography), and the extreme range of the exponent of the electric force appears to be from 1.3 to 2.7. The great mass of the evidence, however, centers about the value 2 (see Fig. 41). It is difficult to account for the values lying below 2, but those lying above permit quite probable explanation. As already noted, most dielectrics contain some moisture and probably possess some electrolytic conductivity. The latter decreases with increasing voltage and the former causes a conductivity increasing with the voltage; each would cause a corresponding negative or positive increment to a normal absorption loss varying as E^2. The total loss is extremely sensitive to temperature change, and as the loss increases the temperature, this doubtless accounts for many of the higher values of the exponent. In built-up insulation, internal ionization also adds an additional loss, and it has been shown that the prevention of this loss reduces the exponent from higher values toward the value 2. (See Fig. 42, showing the variation of loss with voltage in the paper and mica insulation of a high voltage armature conductor.) Two curves are shown, one in the normal state and one with the insulation compressed. In the former the loss increases more rapidly than as the square of the voltage; in the latter more nearly as the square of the voltage. One striking exception should be noted in the work of M. A. Frigon on impregnated paper, which shows the exponent decreasing from 2.7 at 15° C. to 1.9 at 118° C.; this work should be confirmed, as there is no obvious explanation in the light of the above theories.

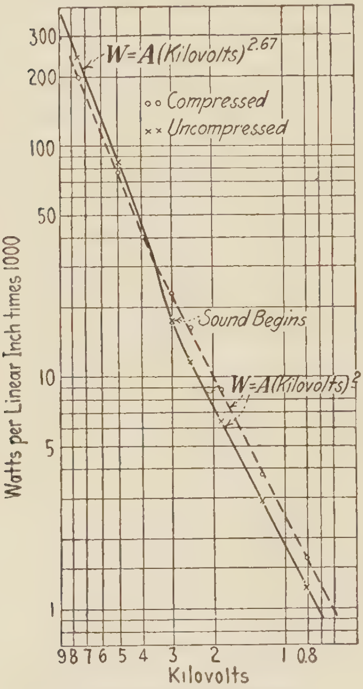


FIG. 42.—Variation of dielectric loss with voltage. Paper and mica.

Loss-frequency Curve.—In the matter of frequency the several theoretical developments are not in entire accord. Wagner shows the loss proportional to both frequency and phase difference, and as the latter first increases and then decreases with the frequency, a uniform variation in the loss is not always to be expected. Lahouse deduces a loss per cycle inversely proportional to the frequency and thus a loss per second independent of the frequency, for all but very low values. The experiments of Granier show a loss per cycle decreasing with the frequency up to 150 cycles, and a loss per second increasing only as the .5th and .6th power of the frequency, thus seeming to lend some force to Lahouse's conclusion. M. A. Frigon reports the losses in impregnated paper increasing nearly in proportion to the frequency between 20 and 60 cycles. H. J. MacLeod, working on such good insulators as glass, pyrex, paraffin, ceresin, and mica and with unusually careful conditions, finds the losses varying as the .85th to .9th power of the frequency. Many other observers have reported losses increasing with the frequency but usually in less than direct proportion. Addenbroke reports three types of variation: (*a*) direct proportion, to be found only in the best insulators such as crown glass and gutta-percha; (*b*) loss independent of frequency, for insulators containing much moisture; and (*c*) a relation lying between these two, to be found in materials having low insulation resistance, *e.g.*, celluloid.

On the other hand, Wagner has examined the variation of the power factor with frequency over a wide range, and finds that the maximum value of the phase difference may occur for different substances in the range of 4 to 1,000 cyles per second, and higher. This means that within the commercial range and far beyond, the phase difference may either increase or decrease with the frequency. This is probably the chief cause of the general confusion to be found in attempting to coordinate the results of different observers, and the reason why no simple empirical law has appeared expressing the influence of frequency on dielectric loss. Wagner has studied the combined influence of temperature and frequency on the phase difference, and finds that temperature variation provides a very convenient means for bringing out the comparatively slow frequency change. Using this method he has reported some remarkable experimental checks with the theoretical expressions already given. It will be seen, therefore, that a general simple relation between loss and frequency is not to be expected, and observations at different frequencies can be interpreted only through knowledge of other properties such as phase difference, temperature, and moisture content.

Loss-temperature Curve.—The influence of temperature on dielectric loss is very great, the loss rapidly increasing with the temperature. But here, too, it does not appear possible to expect a definite law owing to the indirect influence of temperature on other properties. For solids in which the temperature has no marked effect on the structure, *e.g.*, glass, the losses are apparently proportional to some power (from third to fourth) of the absolute temperature, and leakage is still negligible. Substances about to melt but still pasty show large increase in loss due to absorption but not that due to leakage. Passing into the liquid state, the losses due to absorption are transformed into those due to conductance. These facts are very significant in their suggestion that the influence of temperature is chiefly that of increasing the mobility of electric charges or ions in the dielectrics, first in limited paths and afterwards in liquid state in paths quite unrestricted. This view is still further borne out by the behavior of impregnated paper, which does not pass over into the conducting state when the compound is liquefied, the paper acting as a barrier to the passage of the ions.

The only considerable attempt to account for the influence of temperature from a theoretical standpoint is that of Wagner. He attributes the influence of temperature to its effect on the time constant of the material, *i.e.*, the factor multiplying t, the time, in the exponential variation of the anomalous charging current. This enables him to extend the influence of temperature into the alternating case, in its effect on phase difference, and so on the loss. He reports some remarkable general agreements with the results of observation, and his work stands out as a striking support, in the matter of temperature, of Maxwell's theory. An account of this work was given in the foregoing lecture.

With this brief survey of the phenomena arising when dielectrics are subjected to alternating stress, we must bring to a conclusion our review and discussion of the various theories proposed to explain the anomalous behavior of dielectrics. We have seen that, so far as the phenomena under alternating stress are concerned, it is certain that we may trace their origin to dielectric absorption. Consequently the explanation of dielectric absorption stands out as the great unsolved problem in our efforts to control and to design insulation for electric circuits and machinery. This problem has attracted the efforts of both physicists and engineers. Various theories have been advanced to explain the origin and characteristics of dielectric absorption. Unfortunately it must be admitted that no single one of them admits classification as a theory. All of them are rather in the class of hypotheses. This is due to the fact that no one of these so-called theories is subject to exact quantitative test. Perhaps the simplest of them is the theory of Maxwell; yet so far one has not been able to find a simple dielectric showing specific inductive capacity and conductivity alone. Until at least two such simple dielectrics are found, an exact test of Maxwell's theory would seem to be impossible.

On the other hand, we have a large number of qualitative agreements with the several theories, as already briefly reviewed in the foregoing lectures. Some of these are very striking, and until we have a deeper knowledge of the fundamental structure of dielectrics, we must continue to speak of dielectric phenomena in terms of the theories which are thus partially supported. This is a day of experimental research, and it is probable that the near future will enable us to make a closer choice among the numerous proposals which have been made for the solution of this baffling question. It appears probable that the answer will be neither simple nor clear cut. Dielectric absorption will probably ultimately be shown to be complex in character, having its origins in several causes rather than in a single one. In the meantime it behooves us to keep before us as clearly as possible the essential facts which have already been discovered, so that we may make conservative use of them as engineers, and so that we may give approximate weight to each new bit of experimental evidence as it is brought forward. It is with the hope that I may make this task somewhat easier that I have ventured to prepare this brief course of lectures.

SUMMARY AND CONCLUSIONS

We bring to a conclusion this portion of our lectures with the following summary and conclusions:

1. Dielectric absorption is a conspicuous phenomenon, but it is little understood and its origins are obscure. Its general character is well known, as indicated by the decay of the charging current with time, residual charge, etc. On the other hand,

empirical laws of exact form governing this phenomenon are still lacking.

2. Only solids show the complete absorption phenomena of charge and discharge. Liquids often show an apparent absorption in charging but no residual phenomena. Nearly all solid dielectrics show some absorption. In some substances in a very pure state, *e.g.*, sulphur, quartz, paraffin, it is very small, if not negligible, in amount.

3. Large changes in the absorption in solids may be caused by extremely small changes in composition. Impurities and moisture in very small amounts may cause large changes in absorption.

4. The charging absorption current merges into a final steady condition current. Both are strongly increased by increase of temperature, the absorption finally disappearing or changing into conduction.

5. The alternating losses in solid dielectrics are due almost entirely to absorption. This is shown by theoretical analysis, and confirmed by experiment. The losses due to conductivity are usually very small compared with those due to absorption, and there is no evidence of losses of other types. There is nothing to indicate a hysteresis loss of the character pertaining to magnetic materials.

6. Theories of the ultimate nature of the phenomenon of absorption are: (*a*) that it arises in the mixture of two or more dielectrics and depends only on the known quantities, conductivity and specific inductive capacity—the theory of Maxwell; (*b*) that it is due to an anomalous relation between electric displacement and electric force, the seat of which is within the molecule—the theory of Pellat; (*c*) that it may be explained by Lorentz' theory of electron motion within the structure of the atom—the theory of Décombe; (*d*) that it is due to anomalous conductivity and to water in capillaries or interstices in the body of the dielectric.

7. There is an increasing tendency to attribute dielectric absorption entirely to one or more forms of anomalous conductivity. A number of suggestive studies indicate the movement of ions within the body of the dielectric. There is little evidence of definite conductivity as proposed by Maxwell. There is much evidence that such conductivity as is observed is either electrolytic in character, or approaches that to be found in gases. It is very difficult to separate these various types of conductivity, and

it appears not improbable that each of them may play a part. Contemporary research is showing a preference for this field of investigation.

From the standpoint of engineering practice we may say generally that absorption is a conduction phenomenon and, therefore, in all cases of alternating stress occasions a component of current in phase with the applied voltage. This means an energy loss in the dielectric. In the field of communication by telegraphy and telephony these losses cause attenuation, damping, and the confusion of signals and speech. In the field of high-voltage transmission the loss causes heating and danger to the life of insulation. In both cases the situation is aggravated by the fact that composite insulation is used in nearly all such cases and insulation of this type always possesses high absorption. It is remarkable that in the manufacture of high voltage insulation so little attention has been given the property of dielectric absorption. It is certain that the characteristics of commercial insulation are subject to close control by a study of the properties of single materials and their behavior in combination. Research in this direction is certain to produce new knowledge leading to improvements and economies.

In the succeeding lecture I shall describe some recent experimental research bearing on the properties and control of commercial insulation, and shall review some recent discussion as to the nature of the breakdown of insulation under electric stress.

LECTURE IX

PROPERTIES OF HIGH VOLTAGE INSULATION

We have seen that the fundamental laws of even the purest and simplest of dielectrics are not clearly defined. The insulation of all electric circuits and machinery is composed of dielectrics of various types and in various degrees of combination. It is natural, therefore, that it is difficult to control the properties of insulation and impossible to design it with the same degree of accuracy as that pertaining to other elements of the electric system or machine.

In long-distance communication circuits, such as telegraph and telephone cables, dielectric absorption causes large and sustained charging currents, increased attenuation, distorted signals and speech, and limited speeds of transmission. These difficulties have led to many careful studies of the fundamental properties of dielectrics and to the selection of those having low absorption and loss. The selection and control of gutta-percha for use in submarine cables is an example.

It is in the case of high voltage insulation, however, that the greatest difficulties are encountered. Such insulation is always of limited thickness and therefore subject to high electric stress. The dielectric losses cause internal heat which raises the temperature. This limits the current capacity, causes local hot spots and deterioration of the dielectric material, and leads ultimately to failure. These troubles are especially noticeable in the fibrous, flexible insulations of high voltage machines, transformers, and underground cables.

High power factor is earnestly sought after in transmission, distribution, and all station loading. Low power factor is just as earnestly sought in the case of insulation, for the higher its value, the greater the internal loss, the higher the temperature, the lower the current capacity, the shorter the life. The term "power factor" in this case describes a property of the material, and as such is of quite different character and significance from its older connotation. It is no doubt largely for this reason that the custom has arisen of describing this property of dielectrics, not as the cosine of the angle of advance of the charging current over the applied voltage, but as the sine of the difference between that angle and 90 electrical degrees. This angular difference is known as the "phase difference" or "phase defect" and its usage has the added advantage that up to about 2 degrees, the phase difference, itself, in radians, its sine, its tangent, and obviously the power factor in its usual sense, all have the same value within a very small fraction of a per cent. This makes it possible to use these several quantities indiscriminately, thereby greatly simplifying many computations. The use of the tangent of the phase difference is especially convenient, as the ratio of the in-phase to the wattless components of the current.

In the field of power transmission and utilization, attention was first focused on dielectric phase difference by the observations of Siemens, in 1864, on the heating of condensers. This is an early date in the history of electrical engineering, and for many years thereafter the losses in dielectrics under alternating stress received the attention of physicists rather than of engineers. It was in this early period also that residual charge, discovered in the Leyden jar in 1746, was still further stimulating the interest of physicists in dielectric phenomena. This interest showed its first fruits in the brilliant experiments of Hopkinson of the phenomenon of dielectric absorption. It is not generally realized that since Maxwell's time it has been recognized by engineer physicists, such as Rowland, Hess, and a few others, that dielectric absorption necessarily causes alternating dielectric loss, *i.e.*, dielectric phase difference. Engineers became immediately concerned in the value of dielectric phase difference with the first upward step in transmission voltages in the early 'nineties, and with the use of cables for transmission. The problem of the limitation of dielectric phase difference thereafter was clear cut and has been with us ever since. Obviously the applications in which it assumes its greatest importance are the high voltage cable and the commercial power condenser, but its presence and behavior must always be borne in mind in connection with all high voltage insulation, especially that of composite or flexible character.

Although, as stated above, phase difference is a property of the material of the dielectric, and although this has been recognized for many years, and although an abundant literature is replete with experimental observations, it is astonishing to find that our knowledge even of the values to be assigned to particular materials is extremely indefinite, and that little has been accomplished towards a systematic understanding and regulation of the factors which control the values of phase difference. This is largely due to the fact that there are at least four and possibly more different classes of phenomena in dielectrics, any one of which will cause an angle of phase difference; and to the further fact that probably all these causes follow different laws under the influence of varying temperature, frequency, and electric intensity. Each of the following well-recognized properties or conditions of insulation, if present, will cause a dielectric phase difference; (1) normal conductivity, (2) dielectric absorption, (3) anomalous conductivity, (4) absorbed moisture, (5) dielectric hysteresis, and (6) gaseous ionization.

Normal Conductivity.—Under the original theory of Faraday and Maxwell, still regarded by physicists as fundamental in spite of obvious insufficiency, dielectrics are treated as being perfect of their kind, *i.e.*, as possessing specific inductive capacity only. Maxwell, however, recognized that no such perfect solid dielectric exists, and he treated at considerable length the properties of dielectrics which also possess conductivity. We have then in conductivity one of the fundamental causes of dielectric phase difference. It is usually assumed that conductivity contributes a negligible proportion of the losses in dielectrics. This is undoubtedly true for most pure and simple materials, especially at ordinary temperatures.

Although the conductivity of impregnated paper, such as used in high voltage cables, may increase forty times within the range of temperature 25 to 80° C., the conductivity even at the higher temperature accounts for only a fraction of 1 per cent of the total losses. These statements apply to paper which has been well dried and evacuated before impregnation. The presence of moisture in the paper causes a large increase in the conductivity. Large quantities of moisture must be present, however, before the measured conductivity will account for the total loss under alternating voltage.

Moisture.—Many insulating materials, particularly those of porous and fibrous character, absorb moisture readily from the air. Their conductivity is very often greatly increased thereby. In many cases the moisture is taken up quite rapidly and is completely driven off again only with considerable difficulty. As a consequence, exact statements as to the influence of moisture on conductivity and power factor are not possible. Several definite qualitative relations stand out, however, and the behavior of cable paper in this regard may be taken as generally characteristic. Cable paper when standing in the open will absorb from 10 to 15 per cent of weight of moisture. In practice this moisture is driven off by heating at reduced air pressure. I have recently made a study of the changes in the electrical properties of wood-pulp paper through the usual processes of drying. In these experiments the samples consisted of brass tubes 2.54 centimeters in diameter and 122 centimeters long, with twenty-five layers of paper 2.54 centimeters wide wrapped around the tube spirally in the manner used in the construction of high voltage cables. An electrode of sheet lead was applied over the paper, with guard electrodes on either side. Provision was made for measuring the current-time characteristics of a number of these samples, at voltages up to 1,500 volts continuous, and at constant temperature.

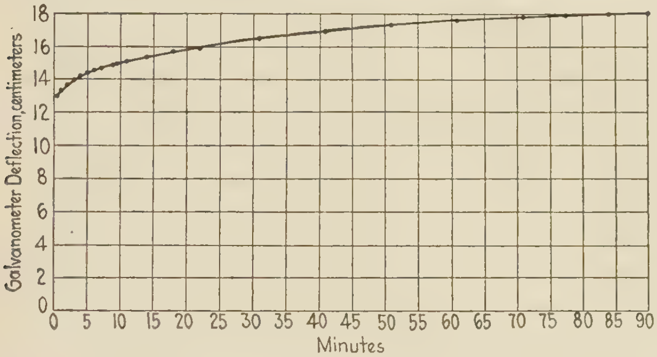


FIG. 43.—The charging current of cable paper containing moisture.

The current through the insulation to the test electrode at 110 volts continuous, and at 20° C. was as shown in Fig. 43. The current rises continuously for 1½ hours after the first application of voltage, and is not yet constant. After discharging and charging a second time at the same temperature, the second curve begins abruptly at approximately the value at which the first curve left off. Much the same effect is observed for a further period of short circuit and charge. After reversal of polarity, the current starts at a lower value and seems to decrease slowly (see Fig. 44). These results indicate that the continuous application of voltage gives a progressive increase in conductivity. The paper thus shows the conspicuous presence of the well-known Evershed effect. All three samples in the group give approximately the the same shape of curves, but their ordinates differ amongst themselves. At this temperature (20° C.) there is only an extremely small indication of residual charge for any of the samples, the reading at 30 seconds on discharge being 5 millimeter galvanometer deflection, which may be compared with the corresponding deflection of 40 centimeters on charge.

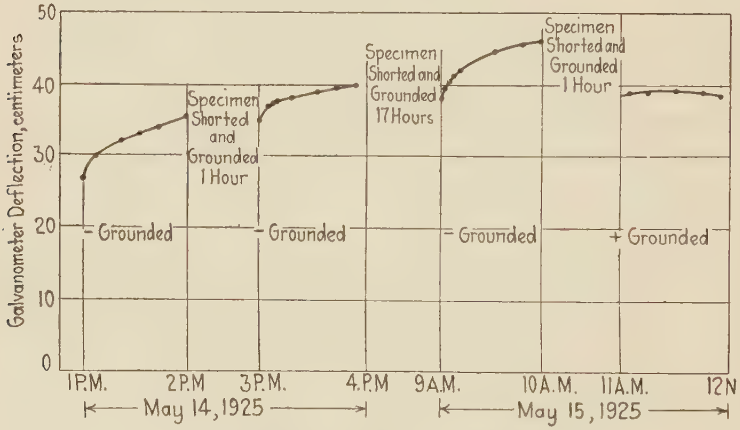


Fig. 44.—Successive charges of moist cable paper.

The temperature of the samples was then raised in steps of approximately 10° C. and allowed to come to a steady state at each temperature. The changes in the electrical characteristics follow very closely the changes in temperature. During the process of temperature change it is easily possible to follow the change in the value of the charging current (Fig. 45). Up to 65° C. there is a steady and rapid increase of the values of the currents, the curves, however, tending to become flatter (see Figs. 46 and 47). The value of the residual charge also increases through this range, the 30-second value reaching 9.2 centimeters at 55° C. It is still very small as compared with the charging current at the corresponding interval, and absorption is as yet not great enough to show itself in the form of the charging-current curve. The samples were allowed to stand overnight at 75° C. It was then found that the charging-current curve was nearly flat and considerably below that at 65° C. Thus, in this temperature region, time enters as a factor. Somewhere between 65 and 75° C. the conductivity of the sample stops rising and decreases (see Fig. 47). The curve of current on discharge is also correspondingly lower, thus indicating a relation between

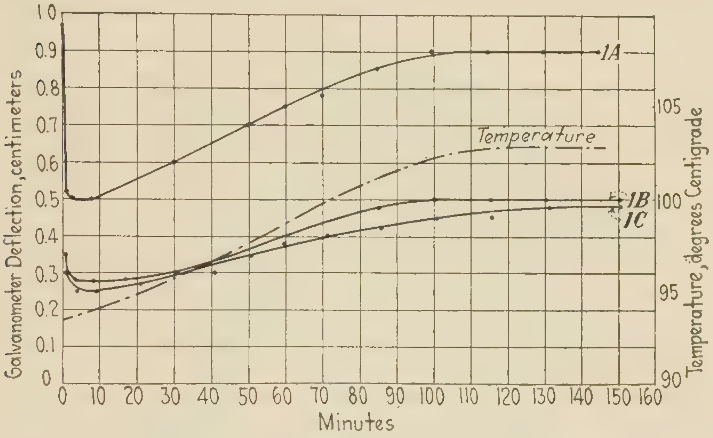
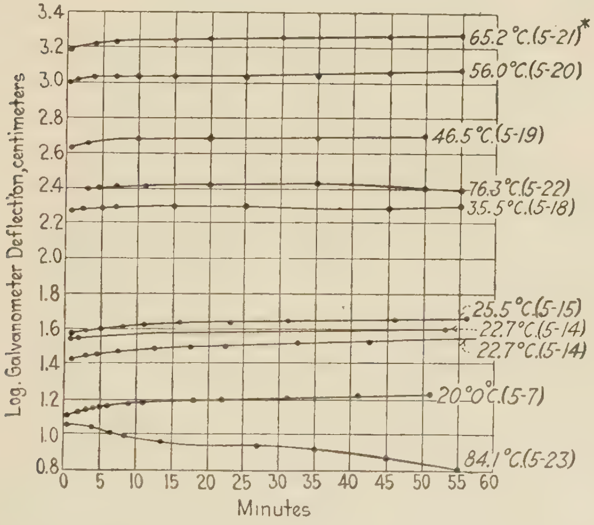


FIG. 45.—The influence of temperature on the conductivity of cable paper.



*(5-21) indicates date e.i. May 21st.

FIG. 46.—The influence of temperature on the conductivity of cable paper.

absorption and moisture content. It should be noted that the absorption is of negligible magnitude as compared with conduction, up to 75° C.

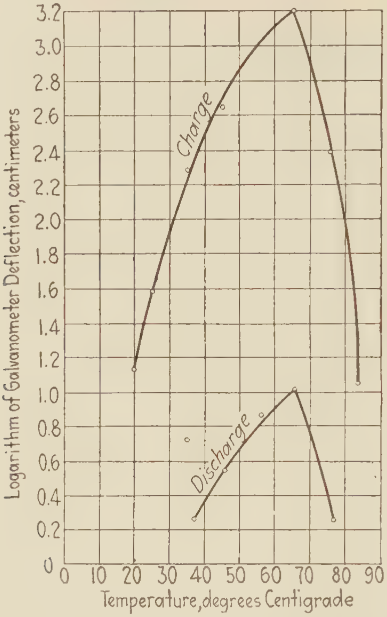


FIG. 47.—The currents of charge and discharge of cable paper first increase and then decrease with increasing temperature.

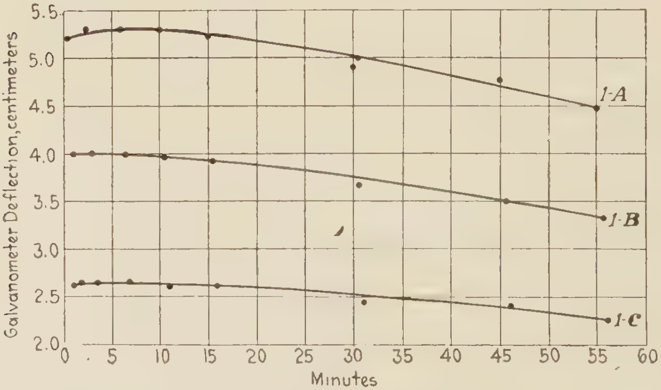


FIG. 48.—The conductivity of cable paper at 85° C.

At 85° C. the charging-current curves rise slightly at the beginning and then fall off (see Fig. 48). The curves are still quite flat, but the decrease, although slow, seems to indicate that absorption begins to play its part in the shape of the curve. After standing overnight at 90° C. the initial rise in the charging-current curve disappears and a typical absorption curve takes its place. In this condition, however, the paper is still extremely sensitive to temperature change. For example, starting at 94° C., the initial limb of the absorption curve is readily observed, but as the temperature is gradually raised to 103° C. over a period of 2½ hours, the current is seen (Fig. 45) to rise and become steady contemporaneously with the temperature. For temperatures above 104° C. the curves are all of typical absorption type (see Fig. 49) with little change in shape up to 125° C. Both absorption and final conductivity continue to decrease in this range suggesting the continued elimination of moisture. Above 85° C. measurements were made at 500, 1,200, and 1,500 volts continuous. In all cases the galvanometer gave deflections in proportion to the voltage, and the shapes of all the curves remained the same.

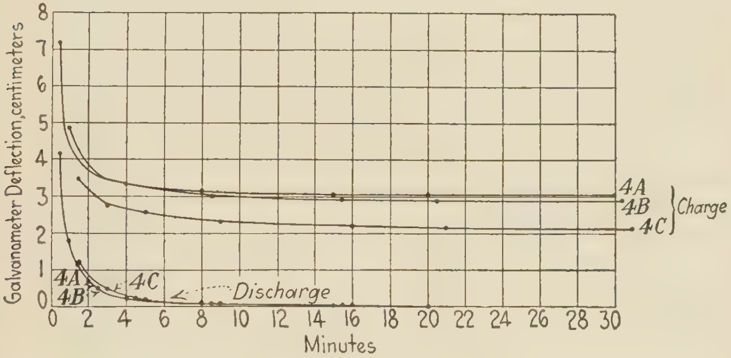


Fig. 49.—Absorption in cable paper at 110° C.

The general conclusions from these studies is that the paper contains large amounts of moisture which is driven off rapidly at 75° C. or above. Up to this point its conductivity masks the usual dielectric properties. At 105° C. the paper seems to reach a fairly definite condition. It appears as a dielectric having marked absorption but relatively high resistivity. On further elevation of temperature, more moisture is driven off with consequent improvement of dielectric properties, although the changes are neither as marked nor as rapid as in the earlier stages. The properties are quite definite at any one temperature, although there are differences of from 50 to 100 per cent as amongst successive samples tested. Characteristic curves of one group at a temperature of 110.6° C. are shown in Fig. 49. The final current of specimen 4-*C* indicates a resistivity of 1.12×10^{15} ohms per cm.^{3}.

In preparing the samples for impregnation they were maintained at a temperature of 155° C. in the drying chamber until they reached a steady state as regards absorption and conduction. Immediately after impregnation at 115° C. the absorption current values were found to have increased from forty to seventy times depending upon the sample, although the relative positions of the three curves of the samples of each set remained about the same. The condition is shown in Fig. 50. In this case the increase in absorption current after impregnation is only about fourteen times. This is due to the lower temperature (82° C.) at which the measurements were taken after impregnation. These increases in absorption and in conductivity decrease slowly with time if the sample is maintained at a high temperature. There is also some evidence that the application of alternating voltage causes further reductions. The samples apparently reach a uniform condition after one or two days of test. These changes offer an interesting problem for future study.

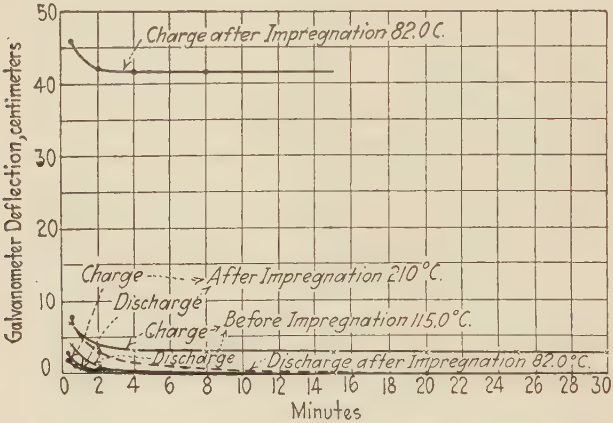


FIG. 50.—Curves of charge and discharge of cable paper, before and after impregnation.

The large increase in the charging current after impregnation is almost entirely due to the increased conductivity. Figures

51 and 52 show the rapid increase after impregnation of both the final and the 1-minute galvanometer deflections with increasing temperature. Obviously the final deflections are proportional to the conduction current, and it will be seen that the ½- and the 1-minute deflections increase at much the same rate. Furthermore, the residual charge, as indicated by the discharge curves, shows relatively a much smaller increase than the conduction current. This is shown in Fig. 50 in which the discharge curve after impregnation and at 82° C. is seen to lie only slightly higher than the curve for 21° C. before impregnation. Figure 53 shows for various temperatures the difference between the deflections taken at intervals during the charging period and the final steady deflection. These curves, therefore, show the total current minus the conduction current, that is to say, the so-called reversible anomalous current. It will be seen that with the exception of the curve at 80° C. the absorption increases with the temperature throughout. The inset on Fig. 53 compares the reversible anomalous current on charge with the current on discharge, for temperatures of 80 and 57° C. It will be seen that at each temperature the two curves are closely coincident, which again indicates that the large increase in the charging current curves after impregnation is due to the increased conductivity caused by the presence of impregnating material. Moreover, this seems to be a uniform conductivity, as that portion of the charging current curve due to absorption is completely reversible.

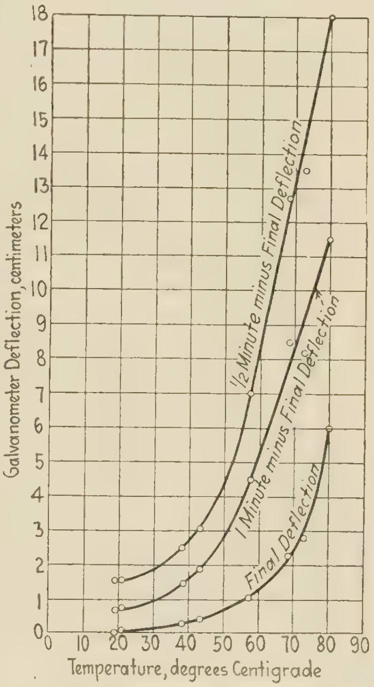


FIG. 51.—Increase of absorption and final conductivity with temperature, in impregnated paper.

The conclusions from these studies is that the influence of moisture on the phase difference on insulating materials is very complex. The variation of electrical properties with both voltage and time noted for continuous voltages is often to be found also in the alternating case. Increase in the amount of absorbed moisture shows itself almost immediately in increased values of phase difference and loss. As a consequence, every effort is made in the manufacture of commercial insulation to exclude

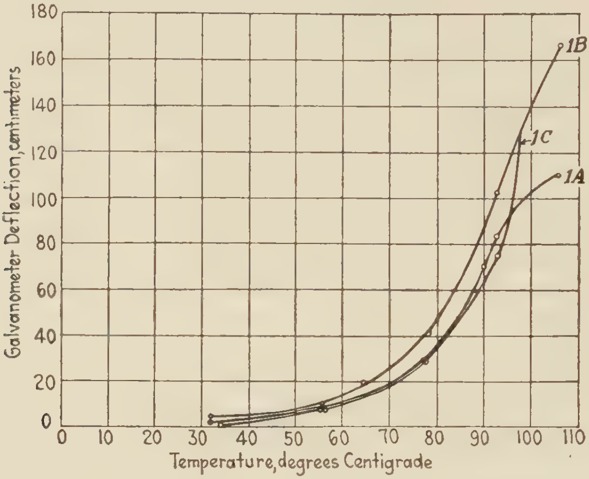


FIG. 52.—Increase of absorption and final conductivity with temperature in impregnated paper.

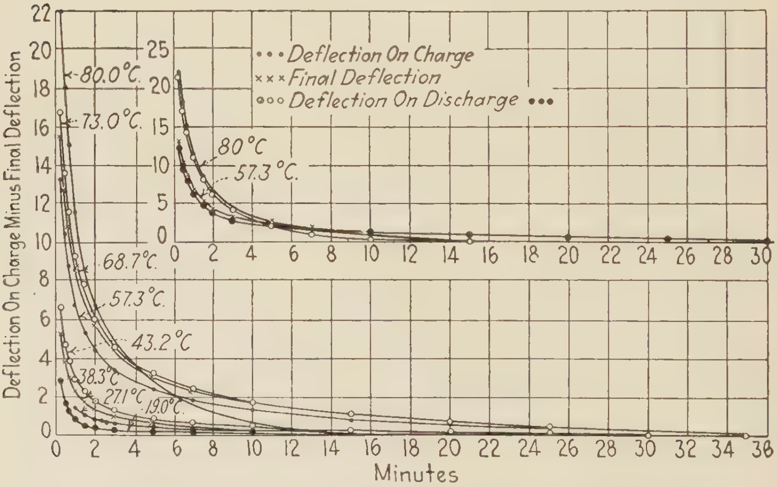


FIG. 53.—The increase in the charging current of cable paper after impregnation is due to the conductivity of the impregnating material.

moisture as completely as possible. It is safe to say, however, that in few cases is it possible completely to eliminate moisture, and that observed values of phase difference and loss are always in some measure increased by residual moisture.

We note further that the conductivity of cable paper is greatly increased on impregnation. There is also a corresponding increase in the dielectric absorption. The increased conductivity falls off rapidly with the temperature and at 20° C. approaches that of the dry unimpregnated paper. The conductivity thus introduced by the compound seems to be constant in character at any one temperature and to possess the irreversible character often observed in liquid dielectrics.

Dielectric Absorption.—Probably the most important of all causes of dielectric phase difference is dielectric absorption, *i.e.*, the phenomenon of after charge and residual charge. It is obvious that if, under applied continuous voltage, current continues to flow over a period of time into or through a dielectric, then under alternating voltage there will flow a component of current in phase with the voltage, which, of course, means a definite angle of phase difference. Absorbent dielectrics, in effect, have for a short interval after the application of voltage a greatly increased value of apparent conductivity. On short circuit a charge continues to flow out long after the electrostatic charge has disappeared, and on reversal of the applied voltage the high initial conductivity appears again. Thus, for the rapid reversals of alternating voltage, the dielectric behaves as though this increased conductivity were continuously present.

Absorption usually occurs whenever a dielectric is composed of two or more different materials. Further, it appears that a very small proportion of a foreign material may cause a large absorption effect. Thus, the absorption often observed in many supposedly pure, simple materials is usually attributed to impurities. This is one of the obscure questions in our imperfect knowledge of dielectric absorption. Although it has been known for years that absorption is one of the most important causes of phase difference, very little attention has been given to the problem of controlling its value and to the study of the behavior of various dielectric materials singly and in combination. That such study would prove profitable is indicated by the fact that a few composite dielectrics have very low values of phase difference and of absorption. These materials apparently are few in number, and their properties obtained only by cut-and-try and by great care in preparation. There is no reason, however, why a further study and control of composite materials should not lead to dielectrics having not only low phase difference, but also other desirable thermal and mechanical properties.

The samples which were used for investigating the influence of moisture in impregnated paper, as described in the foregoing paragraphs, were also studied as regards their behavior under alternating voltage. For various temperatures between 25 and 80° C. the power factor or phase difference was measured at different values of voltage. Figure 50 shows a typical absorption curve as taken with 1,500 volts continuous after the sample had been dried by heating and evacuation, and then impregnated.

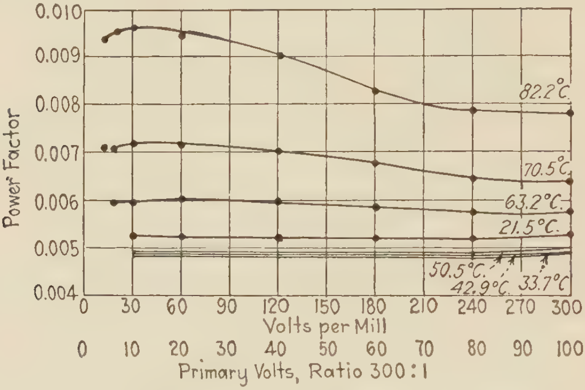


FIG. 54.—Power-factor-voltage relations at different temperatures, in impregnated paper.

Figure 54 shows the power factor voltage curves at different temperature and Fig. 55 shows the power factor temperature and the absorption temperature relations.

These curves, particularly those in Fig. 55, show in very striking manner the close relationship between the power factor and the dielectric absorption. This immediately suggests that it should be possible to predetermine and to control the properties of high voltage insulation by independent studies of its dielectric absorption under continuous potential.

The measurements referred to were made with a Schering bridge, as shown in Fig. 56. This is a well-known method, but I believe that the system of electrostatic screening indicated will be of interest. In our final arrangement we were able to make

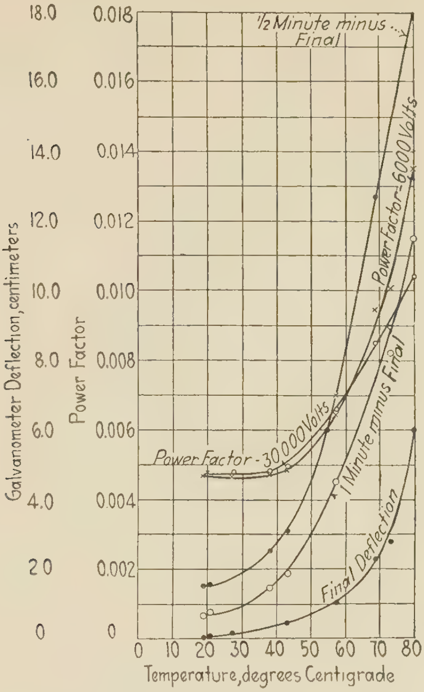


FIG. 55.—Close relationship between power factor and dielectric absorption. Impregnated paper.

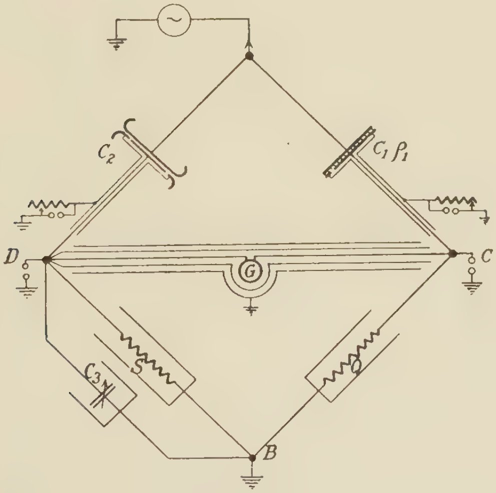


FIG. 56.—Schering bridge with electrostatic screening.

power factor measurements to two significant figures in the fifth decimal place, for power factors between 0.00400 and 0.0070. In order to reach this refinement of accuracy, we find that the system of screening indicated is absolutely necessary.

Anomalous Conduction.—Nearly all liquid dielectrics show some conductivity. Moreover, this conductivity generally varies with both time and voltage. On the application of continuous voltage, the resulting current decreases, approaching a constant value. With increasing voltage the final constant values generally show a departure from Ohm's law. Many attempts have been made to coordinate the results of investigation of this property but without great success. This conductivity is undoubtedly ionic in character and in some cases follows closely the known laws of the ionic conductivity of gases. Values have been obtained for the mobilities of both positive and negative ions for particular materials, but in general the results of such investigations are far from definite in character; this is attributed to the presence of ions or molecular aggregates of different size. Phenomena of this character seem to be particularly susceptible to the presence of impurities in small amounts, to traces of water, etc.

Electrolytic dissociation and resulting conductivity is known to exist in some complex insulating materials. Glass is a remarkable example of this. The metallic constituents of some glasses may be separated out under continuous voltage and deposited on electrodes. Here is an instance of a rigid insulating material for which it is known beyond a doubt that electrolytic ions pass through it from one electrode to another.

As regards the influence of these two types of conductivity on the phase difference of commercial insulation, it may be said that very little is definitely known. There would appear to be good possibility of the presence of the former type, and it is probably true that the residual conductivities of composite dielectrics, made up originally with liquid binders, are in considerable proportion due to the motion of ions of the type usually to be found in liquids.

Hysteresis.—Since the days of Siemens, who first noted the heating of impregnated paper under alternating stress, it has been customary to attribute the losses in dielectrics to some form of molecular friction, apparently arising in the same types of cause pertaining to the case of magnetic hysteresis. There are many differences between the two phenomena which indicate that they are of essentially different character, and it is therefore undesirable to use the word hysteresis in connection with dielectric behavior. There are many other factors, as enumerated above, that appear to be quite sufficient to account for all of the losses which are observed. It is, of course, possible that in addition to these well-recognized causes for loss there may be some residual type of loss arising in the orientation or deformation of molecules and atoms. In fact, there have been numerous suggestions that the phenomenon of absorption itself arises in frictional deformation or the motion of electrons within the atom itself. With due weight given to these considerations, it may still be said that the evidence that the nature of the losses in dielectrics is of the character usually understood by the word hysteresis is so small as to make it appear very unlikely.

Gaseous Ionization.—Many forms of commercial insulation are composite in character and built up in layers. Conspicuous examples are the insulation of all electromagnetic machinery and of many types of high voltage underground cable. The assembly and application of this type of insulation invariably provides opportunity for the enclosure or entrapping of a certain amount of air which is never completely removed by such processes as evacuation and subsequent pressure impregnation. This air, when voltage is applied to the insulation, breaks down electrically under the process known as ionization. It is also known that in some cases the constituents of impregnating materials break down under stress, with the generation of gases and the further increase in the size of the voids in the body of the insulation. These gases are also subject to ionization. The products of this gaseous ionization are usually highly active oxygen and ozone. Not only is the ionized gas a good conductor, but the products of the ionization attack the surrounding material rendering it conducting, thus further increasing the conductivity.

The high voltage armature bar presents perhaps the most important of all problems of insulation. It is the weakest point of the machine, and the consequences of its failure are probably more costly and more disastrous than in any other case. Owing to the exceptionally high values of voltage gradient occurring in this type of insulation to the laminated form in which it is applied and to the shape of the cross-section of the usual turbo-alternator coil, the conditions are highly favorable for high dielectric losses and for losses due to the breakdown of air layers between the layers of solid insulation.

High voltage armature insulation is often constructed with alternate layers of mica and some sort of flexible material, as for example, impregnated paper or varnished cambric, as a binder. The mica serves a double purpose. It reduces the conductivity of the insulation and therefore the dielectric losses, and further it is a heat resisting material retaining its insulating properties well beyond the temperature at which electrical machinery is usually operated.

Table I.—Influence of Pressure on Loss Due to Internal Corona
Treated Cloth and Mica Group

Bar number	Loss, watts				Total decrease, per cent	Charging currents × 10^3	
	No compression	Compressed				Before compression	After compression
		Start	24 hours	48 hours			
46	0.549	0.544	0.533	......	2.95	1.4	1.41
47	0.586	0.522	.....	0.517	11.8	1.43	1.47
48	0.494	0.479	0.462	......	6.5	1.41	1.44
49	2.00	1.35	1.25	......	37.5	1.40	1.58
50	1.95	1.30	1.30	......	33.3	1.57	1.77
51	0.92	0.88	0.88	......	5.4	1.58	1.62
52	*0.687	0.502	0.502	......	27.0	*0.98	1.05
53	2.14	1.6	1.53	......	28.5	1.5	1.66
54	1.9	1.4	.....	1.37	27.9	1.55	1.65
55	3.13	2.31	.....	2.11	32	1.66	1.9
56	3.32	2.43	.....	(2.26	32	1.71	1.93
57	4.31	3.29	3.03	8 days)	29.7	1.73	2.1
58	4.03	4.05	.....	3.88	3.75	2.24	2.52
59	2.8	2.85	2.9	(72 hours)	(3.5 increase)	2.00	2.16
60	3.96	3.82	3.4	......	14.2	2.11	2.36

Readings for 24 and 48 hours corrected for temperature differences.
* 5,000 volts

In a recent series of experimental studies fifteen sample armature bars were prepared in five groups of three bars each. The three bars in each group were identical as to make-up and assembly. The five groups were constructed with progressively increasing amounts of mica, the cloth binder being so varied as to preserve the same thickness for all the samples. The dielectric loss of each sample was measured at various values of voltage. For this purpose a quadrant electrometer in the well-known wattmeter connection was used. All of the measurements were made at atmospheric temperature. Following these observations a similar set was taken with the insulation on each sample compressed with screw clamps. The purpose of this was to squeeze out as much of the entrained air as possible, thus closing up the air layers and reducing internal gaseous ionization. A summary of the results is given in Table I. In the table the bars in groups of three are in the order of decreasing mica content. The second column gives the loss at normal operating voltage at atmospheric temperature. The three succeding columns give the values to which the loss is reduced by compressing the insulation. The

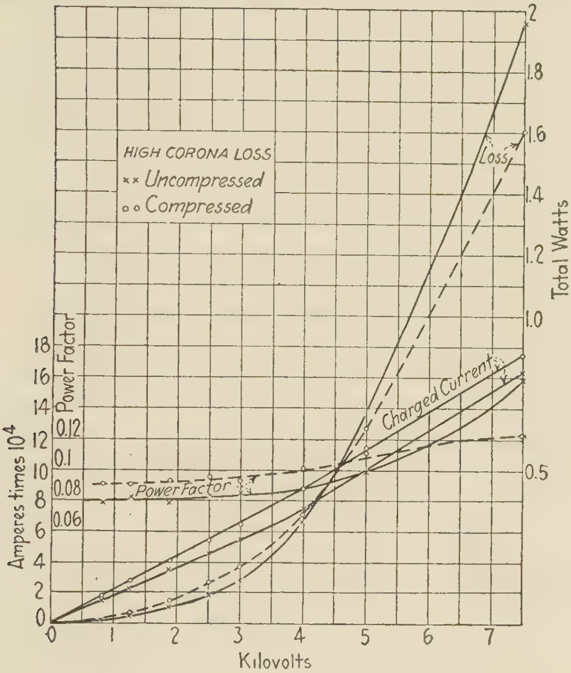


FIG. 57.—Variation of dielectric loss with voltage. Paper and mica.

sixth column gives the percentage decrease of loss caused by compression. The last two columns show that the charging current of the sample in every case is increased by the compression. The variation of the dielectric losses with voltage is indicated in Figs. 57 and 58. The observations are plotted as taken in Fig. 57 and in logarithmic coordinates in Fig. 58. Based on these and other investigations of similar character, it was found that the dielectric loss in this type of insulation increases as the square of the voltage in the lower range, that is to say, up to a potential gradient of about 16,000 volts per centimeter. Above this gradient the loss also increases directly as a power of the voltage, but this power is greater than 2 for good tightly wrapped mica and paper insulation. The exponent in this region may be as low as 2.15. For bars high in mica, well dried and expanded under temperature, the exponent may reach 4.4. The values for the whole series of samples investigated lay between 2.15 and 2.6 with an average value of 2.3. A typical example of the change in the exponent with increasing voltage gradient is shown in Fig. 58. The solid line shows the relation between voltage and loss before the bar is compressed, and plotted in logarithmic coordinates. The broken line gives the results taken after the bar was compressed. It will be noticed that the result of compression is to raise the loss for values of voltage below 3.5 kilovolts and to lower the loss for voltages above 3.5 kilovolts.

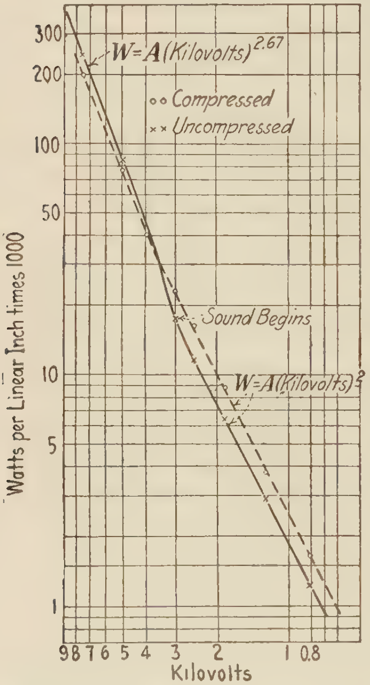


Fig. 58.—Variation of dielectric loss with voltage. Paper and mica.

The increase in loss above 16,000 volts per centimeter is largely due to gaseous ionization in entrapped air spaces. This is shown by the simultaneous beginning of the audible sound of internal gaseous breakdown and the increase in loss; and also by the reduction in each of these when the insulation is compressed. On compression the voltage-loss curves show an increase of loss below 16,000 volts per centimeter and a decrease at higher voltages. Notwithstanding the lower losses in the higher range, the charging current is at the same time increased. In the lower range of voltage, compression increases the voltage gradient and increases the loss. In the upper range compression also diminishes the loss due to gaseous ionization, and this decrease is greater than the increased dielectric loss in the solid insulation. The charging current is little affected by the change in loss owing

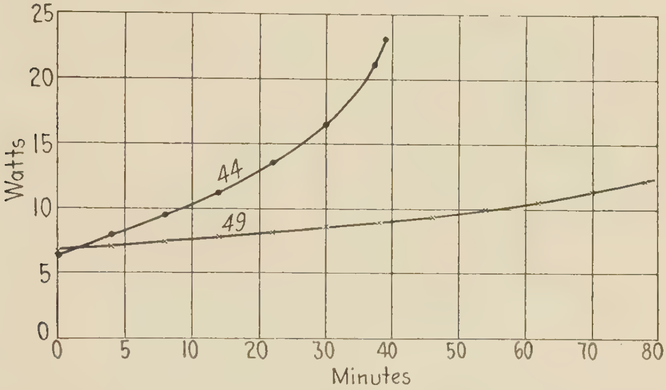


Fig. 59.—Increase of loss due to increase of temperature. Paper and mica.

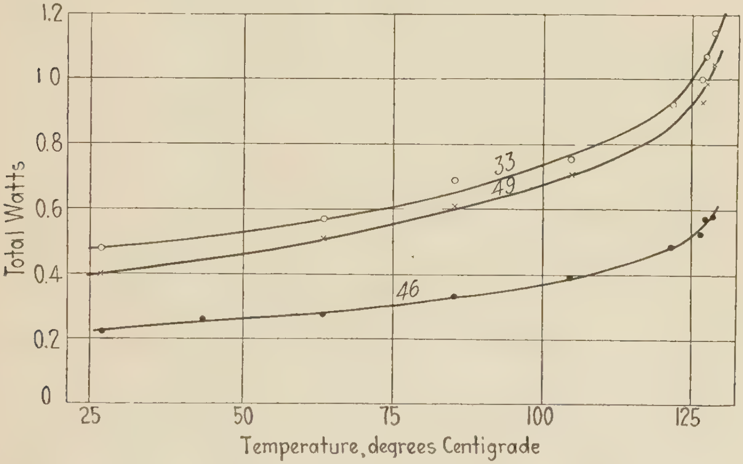


Fig. 60.—Increase of loss due to increase of temperature. Paper and mica.

to the low power factor, and shows a uniform increase over the whole range of voltage when the insulation is compressed. These conclusions are plainly indicated from Figs. 59 and 60. Figure 59 shows for two samples the gradual increase of the loss with time at twice the rated voltage, one sample contained no mica, and the other contained about 80 per cent of the normal content of mica. Figure 60 shows the relation between loss and temperature on three bars of different proportions of mica content. These curves indicate clearly the limitations of the life of this type of insulation as determined by the temperature. They also indicate that the failure of such insulation is largely caused by a progressive increase of internal loss, which causes a local elevation of temperature, which, in its turn, causes a further increase in loss, and so on to rupture.

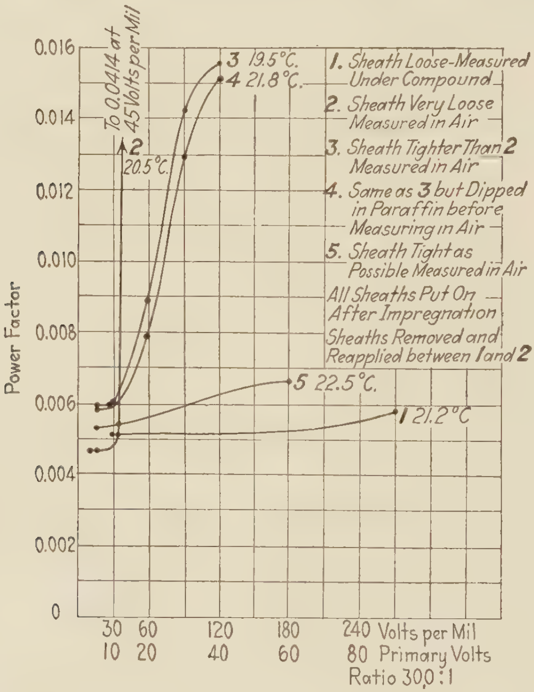


FIG. 61.—Losses due to internal gaseous ionization.

Internal gaseous ionization also occurs in impregnated paper insulated underground cables if the impregnation is not complete and if the lead sheath does not fit closely to the paper insulation.

Figure 61 shows a number of power factor voltage curves on some of our experimental samples in which the outer electrodes were applied with varying degrees of tightness.

It will be seen, therefore, that this type of loss is not necessarily inherent in the material itself but is a result of imperfect methods of assembling and applying the material. It is probable, however, that even at very low stresses this type of loss is present to some extent in all insulations in the class mentioned. Standard specifications for cables, for example, include clauses giving tests for increase in power factor due to this cause and limiting values for such increase, thereby recognizing its necessary presence. Power factor curves of this type of insulation show marked increase only above certain values of stress, and such increase is in all probability rightly attributed to this type of ionization.

It cannot be emphasized too strongly, however, that internal ionization is highly destructive to the structure of composite insulation, and that if such insulation is operated above a stress at which ionization is known to occur, the days or months of the life of this insulation are already numbered.

Bearing in mind these various causes giving rise to a phase difference in dielectrics, it is not to be wondered at that there is much conflict of evidence as to the behavior under different conditions of insulation of any particular type. Of the several causes enumerated, the laws of only two of them may be said to be well known, namely, conduction and absorption. Inherent conductivity of the type found in metals leads to a value of losses readily calculable. Dielectric absorption has been carefully studied and it is known that, in insulation of comparatively simple structure, absorption accounts for most of the loss. In the uncertain laws controlling the losses of other character, there is plenty of explanation of the erratic behavior often reported. It may be pointed out, however, that these uncertain phenomena are usually of a type which seems to yield to care and control in preparation of materials and methods of assembly. Moisture may be largely driven out, anomalous conductivity may be reduced to extremely small proportions, and it is possible to reduce gaseous ionization to negligible proportions. With care of this character, therefore, it is legitimate to consider the behavior of dielectrics from the standpoint of conductivity and absorption alone. It has been shown that the extension of Maxwell's theory of absorption to the alternating case will account for the outstanding features of the behavior of composite insulation. For example, the loss is in proportion to the square of the voltage and to the frequency. Moreover, it is shown that the variation of power factor with the frequency shows a maximum value, which may lie at a low or very high value of frequency, depending upon the constituents of the material. Further, and due to the same causes, variations of temperature within the usual range may cause either sharp increase or sharp decrease of power factor. The fact that variations of this character are in accord with the simple theory of absorption as the principal source of loss is extremely encouraging and a great step in advance.

In closing this brief discussion of the origin of phase difference in dielectrics, it may be pointed out that a moderate value of phase difference or power factor appears to be practically a necessary feature for composite insulation. On the other hand, there is no objection to a phase difference of moderate amount provided that it remains constant. If the phase difference of a given dielectric increases with the electric stress, its cause should be carefully investigated and the insulation always operated below the stress at which such increase in phase difference begins. The increase due to temperature is more difficult to control, but there is reason to suppose that it has a definite value in any particular case, and that it is therefore subject to control. If these considerations are correct, there is good reason to hope that a well-considered program of experimental research will result in the placing of the design of insulation on an engineering basis comparable to that pertaining to the other elements of circuits and machinery.

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V. Phenomena in Revolving Fields

The presence of a positive or a negative twisting couple on a dielectric in a rotating field does not necessarily indicate the presence of dielectric absorption, nor can it be taken as a measure of absorption.

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VI. Theories of Dielectric Absorption

Excellent review of theoretical discussion up to 1913 are given by F. W. Grover, *Bull. Bur. Stand.*, **7**, 495 (1911) and by E. von Schweidler in Graetz' "Handbuch d. Elek. u. Mag.," I, 232 (1918).

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